CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Algebra

TUTORIAL SHEET 1: Group

Q1. Prove that if a group has only four elements then it must be abelian. **[1998]**

Q2. If H and K are subgroups of a group G, then show that HK is a subgroup of G if and only if HK=KH. **[1998]**

Q3. Show that every group of order 15 has normal subgroup of order 5. **[1998]**

Q4. If p is a prime number and $p^{\alpha}|o(G)$, then prove that G has a subgroup of order p^{α}. **[1999]**

Q5. Let n be a fixed point integer and let Z_n be the ring of integers modulo n. Let $G = \{\bar{a} \in Z_n | \bar{a} \neq 0$ and a is relatively prime to $n\}$. Show that G is a group under multiplication defined in Z_n. Hence or otherwise, show that $a^{\emptyset(n)} = a \ (mod\ n)$ for all integers a relatively prime to n where $\emptyset(n)$ denotes the number of positive integers that are less than n and are relatively prime to n. **[2000]**

Q6. Let K be a field and G be a finite subgroup of the multiplicative group of nonzero elements of G. Show that G is a cyclic group. **[2001]**

Q7. Let N be a normal subgroup of a group G. Show that G/N is abelian if and only if for all. $x, y \in G, xyx^{-1}y^{-1} \in N$ **[2001]**

Q8. Show that a group of order 35 is cyclic **[2002]**

Q9. Show that a group of p^2 is abelian, where p is a prime number. **[2002]**

Q10. Prove that a group of order 42 has a normal sub-group of order 7. **[2002]**

Q11. If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G. **[2003]**

Q12. If p is a prime number of the form $4n + 1$, n being a natural number, then show that congruence $x^2 \equiv -1 \bmod p$ is solvable. **[2004]**

Q13. Let G be a group such that of all $a, b, \in G$
(i) $ab = ba$ (ii) $(O(a), O9b)) = 1$ then show that $O(ab) = O\ (a)\ O$ (b) **[2004]**

Q14. If M and N are normal subgroups of a group G such that $\cap N = \{e\}$, show that every element of M commutes with every element of N. **[2005]**

Q15. Let H and K be two subgroups of a finite group G such that $|H| > \sqrt{|G|}$ and $|K| > \sqrt{|G|}$. Prove that $H \cap K \neq \{e\}$ **[2005]**

Q16. Let S be the set of all real numbers except -1. Define * on S by
a*b= a + b + ab
Is $(S, *)$ a group?
Find the solution of the equation
2 * x * 3 = 7 in S **[2006]**

Q17. Let $O(G) = 108$. Show that there exists a normal subgroup or order 27 or 9.

Q18. Let G be the set of all those ordered pairs (a, b) of real numbers for which $a \neq 0$ and define G an operation $\otimes$ as follows:
$(a, b) \otimes (c, d) = (ac, bc + d) \otimes$. If it is a group, is G abelian?
Examine whether G is a group w.r.t the operation $\otimes$. If it is a group, is G abelian? **[2006]**

Q19. If in a group G, $a^5 = e$, e is the identity element of G and $aba^{-1} = b^2$ for $a, b \in G$ then find the order of b. **[2007]**

Q20. (i) Prove that there exists no simple group of order 48.
(ii) $1 + \sqrt{-3}\ in\ Z[\sqrt{-3}]$ is an irreducible element, but not prime. Justify your answer. **[2007]**

Q21. Let R_o be the set of all real numbers except zero. Define binary operation * on R_o as:
a * b = | a | b where | a | denotes absolute value of a.
Does $(R_o, *)$ form a group? Examine. **[2008]**

Q22. Show that the alternating group on four letters A_4 has no subgroup of order 6. **[2009]**

Q23. Let $G = R - \{-1\}$ be the set of all real numbers omitting -1. Define the binary relation $*$ on G by $a * b = a + b + ab$. Show $(G, *)$ is a group and it is abelian. **[2010]**

Q24. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify. **[2010]**

Q25. Show that the set

$$G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$$

Of six transformations on the set of Complex numbers defined by

$$f_1(z) = z, f_2(z) = 1 - z$$
$$f_3(z) = \frac{z}{(z-1)}, f_4(z) = \frac{1}{z}$$
$$f_5(z) = \frac{1}{(1-z)} \text{ and } f_6(z) = \frac{(z-1)}{z}$$

Is a non-abelian group of order 6 w.r.t. composition of mappings. **[2011]**

Q26. Prove that a group of Prime order is abelian **[2011]**

Q27. How many generators are there of the cyclic group $(G, .)$ of order 8? **[2011]**

Q28. Give an example of a group G in which every proper subgroup is cyclic by the group itself is not cyclic. **[2011]**

Q29. Let a and b the element of a group, with $a^2 = e,\ b^6 = e$ and $ab = b^4a$
Find the order of Ab, and express its inverse in each of the forms $a^m b^n$ and $b^m a^n$.
[2011]

Q30. How many elements of order 2 are there in the group of order 16 generated by a and b such that the order of a is 8, the order of b is 2 and $bab^{-1} = a^{-1}$. **[2012]**

Q31. Show that the set of matrices $S = \left\{\begin{pmatrix} a & -b \\ b & a \end{pmatrix} a, b \in R\right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f: C \to S$ defined by $f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here R is the set pf real numbers and C is the set of complex numbers). **[2013]**

Q32. Give an example of an infinite group in which every element has finite order.
[2013]

Q33. What is the maximal possible order of an element in S_{10}? Why? Give an example of such an element. How many elements will there be in S_{10} of that order? **[2013]**

Q34. Let G be the set of all real 2×2 matrices $\begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$ where $xz \neq 0$. Show that G is a group under matrix multiplication. Let N denote the subset $\left\{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R\right\}$. Is N a normal subgroup of G? Justify your answer. **[2014]**

Q35. How many generators are there of the cyclic group G of order 8? Explain. **[2015]**

Q36. Taking a group [e, a, b, c] of order 4, where e is the identity, construct composition tables showing that one is cyclic while the other is not. **[2015]**

Q37. Let p be a prime number and Z_p denotes the addictive groups of integers module p. Show that every non zero element of Z_p generates Z_p **[2016]**

Q38. Let G be a finite group H and K subgroups of G such that K C H. Show that (G:K)= (G:H)(H:K) **[2019]**

Q39. Write down all quotient groups of the group Z_{12} **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 2: Homomorphism of groups

Q1. If $\emptyset$ is a homomorphism of G into $\bar{G}$ with kernel K, then show that K is a normal subgroup of G. **[1999]**

Q2. Let M be a subgroup and N a normal subgroup of a group G. Show that MN is a subgroup of G and MN/N is isomorphic to $M/(M \cap N)$. **[2000]**

Q3. Verify that the set E of the four roots of $x^4 - 1 = 0$ forms a multiplicative group. Also prove that a transformation $T, T(n) = i^n$ is a homomorphism from I_+(Group of all integers with addition) onto E under multiplication. **[2004]**

Q4. If $f : G \to G'$ is an isomorphism, prove that the order of $a \in G$ is equal to the order of $f(a)$. **[2005]**

Q5. If G is a group of real numbers under addition and N is the subgroup of G consisting of integers, prove that G/N is isomorphic to the group H of all complex numbers of absolute value 1 under multiplication. **[2006]**

Q6. Let G and $\bar{G}$ be two groups and let $\emptyset > G \to \bar{G}$ be homomorphism. For any element $a \in G$.

(i) Prove that $0(\emptyset(a))/0(a)$.

(ii) Ker $\emptyset$ is a normal subgroup of G. **[2008]**

Q7. If R is the set of real numbers and R_+ is the set of positive real numbers, show that R under addition $(R, +)$ and R, under multiplication $(R_+, .)$ are isomorphic. Similarly if Q is the set of rational number and Q the set of positive rational numbers, are (Q, +) and $(Q_+, .)$ isomorphic? Justify your answer. **[2009]**

Q8. Determine the number of homomorphisms from the additive group Z_{15} to the additive group Z_{10}. (Z_n is the cyclic group of order n). **[2009]**

Q9. Let $(R^*, .)$ be the multiplicative group of non-zero reals and $(GL.(n, R), X)$ be the multiplicative group of $n \times n$ non singular real matrices. Show that the quotient group $GL\,(n, R)/SL(n, R)$ and $(R\,*.)$ are isomorphic where

$SL(n, R) = \{A \in GL\,(n, R)/\det A = 1\}$

What is the centre of $GL\,(n, R)$? **[2010]**

Q10. How many conjugacy classes does the permutation group S_5 of permutations 5 numbers have? Write down one element in each class (preferably in terms of cycles). **[2012]**

Q11. What are the orders of the following permutations in S_{10}?

$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{bmatrix}$ and $(1, 2, 3, 4, 5)\,(6, 7)$ **[2013]**

Q12. Let G be a group of order n. Show that G is isomorphic to a subgroup of the permutation group S_n **[2017]**

Q13. Show that group $Z_5 \times Z_7$ and Z_{35} are isomorphic **[2017]**

Q14. Show that the quotient group of $(R_1 +)$ module Z is isomorphic to the multiplicative group of complex number on the unit circle in the complex plane. Here R is the set of real number and Z is set of integers. **[2018]**

Q15.If G and H are finite groups whose order are relatively prime, then prove that there is only one homomorphism from G to H, the trivial one **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 3: Ring

Q1 Let $(R, +, .)$ be a system satisfying all the axioms for a ring with unity with the possible exception of $a + b = b + a$. Prove that $(R, +, .)$ is a ring. **[1998]**

Q2 If p is prime then prove that Z_p is a field. Discuss the case when p is not a prime number. **[1998]**

Q3 Let D be a principal ideal domain. Show that every element is neither zero not a unit in D is a product of irreducibles. **[1998]**

Q4. Let R be a commutative ring with unit element whose only ideals are (o) and R itself. Show that R is a field. **[1999]**

Q5. Let F be a finite field. Show that the characteristic of F must be a prime integer p and the number of elements in F must be p^m for some positive integer m. **[2000]**

Q6. Let F be a field and $F[x]$ denote the set of all polynomials defined over F. If $f(x)$ is an irreducible polynomial in $F[x]$, show that the ideal generated by $f(x)$ in $F(x)$ is maximal and $F[x]/(f(x))$ is a field. **[2000]**

Q7. Show that any finite commutative ring with no zero divisors must be a field. **[2000]**

Q8. Prove that the polynomial $1 + x + x^2 + \cdots ... + x^{p-1}$ where p is a prime number, is irreducible over the field of rational numbers. **[2001]**

Q9. If R is a commutative ring with unit element and M is an ideal of R, then show that M is a maximal ideal of R if and only if R/M is a field. **[2001]**

Q10. Prove that every finite extension of a field is an algebraic extension. Give an example to show that the converge is not true. **[2001]**

Q11. Show that the polynomial $25x^4 + 9x^3 + 3x + 3$ is irreducible over the field of rational numbers. **[2002]**

Q12. Prove that in the ring $F[x]$ of polynomial over a field F, the ideal $I = [p(x)]$ is maximal if and only if the polynomial $p(x)$ is irreducible over F. **[2002]**

Q13. Show that every finite integral domain is a field. **[2002]**

Q14. Let F be a field with q elements. Let E be a finite extension of degree n over F. Show that E has q^n elements **[2002]**

Q15. Show that the ring

$$Z[i] = (a + bi \mid a \epsilon Z, b \epsilon Z, i = \sqrt{-1})$$

Of Gaussian integers is a Euclidean domain. **[2003]**

Q16. (i) Let R the ring of all real-valued continuous functions on the closed interval [0, 1]. Let
$M = \{f(x) \epsilon R \mid f(1/3) = 0\}$ show that M is a maximal ideal of R.
(ii) Let M and N be two ideals of a ring R. Show that $M \cup N$ is an ideal of R if and only if either $M \subseteq N$ or $N \subseteq M$. **[2003]**

Q17. (i) Show that $Q(\sqrt{3}, i)$ is the splitting field for $x^5 - 3x^3 + x^2 - 3$ where Q is the field of rational numbers.
(ii) Prove that $x^2 + x + 4$ is irreducible over F, the field of integers modulo 11 and prove further that $\frac{F[x]}{(x^2+x+4)}$ is a field having 121 elements. **[2003]**

Q18. If R is a unique factorization domain (UFD), then prove that $R[x]$ is also a UFD. **[2003]**

Q19. Prove that if the cancellation law holds for a ring R then $a(\neq 0) \in R$ is not a zero divisor and conversely. **[2004]**

Q20. The residue class ring $\frac{Z}{(m)}$ is a field $iff\;\; m$ is a prime integer. **[2004]**

Q21. Define irreducible element and prime element in an integral domain D with units. Prove that every prime element in D is irreducible and converse of this is not (in general) true. **[2004]**

Q22. Show that $(1 + i)$ is a prime element in the ring R of Gaussian integers. **[2005]**

Q23. Prove that any polynomial ring $F[x]$ over a field F is a U.F.D. **[2005]**

Q24. Show that

$$Z[\sqrt{2}] = \{a + \sqrt{2}\, b \mid a, b\, \epsilon\, Z\}$$

Is a Euclidean domain. **[2006]**

Q25. Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $, b, c, d \in Z$. Show that R is a ring under matrix addition and multiplication. Let $A = \left\{\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \mid a, b \in Z\right\}$

Then show that A is a left ideal of R but not a right ideal of R. **[2007]**

Q26. Show that in the ring
$R = \{a + b\sqrt{-5} \mid a, b \in Z\}$
The elements $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d. in R, where $\gamma = 7(1 + 2\sqrt{-5})$ **[2007]**

Q27. Suppose that there is a positive even integer n such that $a^n = a$ for all the elements ‘a’ pf some ring R. Show that $a + a = 0$ for all $a \in R$ and $a + b = 0 \Rightarrow a = b$ for all $a,\ b\ \in R$. **[2008]**

Q28. Let R be a ring with unity. If the product of any two non zero elements is non zero prove that $ab = 1 \Rightarrow ba = 1$. Whether Z_6 has the above property or not explain. Is Z_6 an integral domain? **[2008]**

Q29. Prove that every Integral Domain can be embedded in a field. **[2008]**

Q30. Show that any maximal ideal in the commutative ring F[x] of polynomials over a field F is the principal ideal generated by an irreducible polynomial. **[2008]**

Q31. How many proper, non zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $Z_{12} \oplus Z_{12}$ have? Why? **[2009]**

Q32. Show that Z[X] is a unique factorization domain that is not a principal ideal domain (Z is the ring of integers). Is it possible to give an example of principal ideal domain that is not a unique factorization domain? (Z [X] is the ring of polynomials in the variable X with integer.) **[2009]**

Q33. How many elements does the quotient ring $\frac{Z_5[X]}{(X^2+1)}$ have?
Is it an integral domain? Justify your answers. **[2009]**

Q34. Let $C = \{f: I = [0,1] \to R \mid f \text{ is continous}\}$. Show C is a commutative rign with 1 under pointwise addition and nultiplication. Determine whether C is an integral domain. Explain. **[2010]**

Q35. Consider the polynomial ring Q[x]. Show $p(x) = x^3 - 2$ is irreducible over Q. Let I be the ideal in Q[x] generated by p(x). Then show that $Q[x]/I$ is a field and that each element of it is of the form $a_0 + a_1 t + a_2 t^2$ with a_0, a_1, a_2 in Q and $t = x + 1$. **[2010]**

Q36. Show that the quotient ring $Z[i]/(1 + 3i)$ is isomorphic to the ring Z /10 Z where Z[i] denotes the ring or Gaussian integers. **[2010]**

Q37. Let F be the set of all real valued continuous functions defined on the closed interval [0, 1]. Prove that $(F, +, .)$ is a Commutative Ring with unity with respect to addition and multiplication of functions defined pointwise as below:

and $$\left.\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ (f,g)(x) &= f(x).g(x)\end{aligned}\right\} x \in [0,1]$$

where $f, g \in F$. **[2011]**

Q38. Describe the maximal ideals in the ring of Gaussian integers $Z(i) = \{a + bi \mid a, b \in Z\}$ **[2012]**

Q39. Is the ideal generated by 2 and X in the polynomial ring Z[X] of polynomials in a single variable X with coefficients in the ring of integers Z, a principal ideal? Justify your answer? **[2012]**

Q40. Let $J = \{a + bi \mid a, b \in Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J: Euclidean domain, principal ideal domain, unique factorization domain? Justify your answer **[2013]**

Q41. Let $R^C =$ ring of all real valued continuous functions on [0, 1], under the operations.
$(f + g)x = f(x) + g(x)$
$(fg)x = f(x)g(x)$
Let M= $\{f \in R^C \mid f(1/2) = 0\}$
Is M a maximal ideal of R? Justify your answer. **[2013]**

Q42. Show that Z_7 is a field. Then find $([5]+[6])^{-1}$ and $(-[4])^{-1}$ in Z_7. **[2014]**

Q43. Show that the set $\{a+b\omega : \omega^3 = 1\}$, where a and b are real numbers is a field with respect to usual addition and multiplication. **[2014]**

Q44. Prove that the set $Q(\sqrt{5}) = \{a+b\sqrt{5} : a, b \in Q\}$ is a commutative ring with identity. **[2014]**

Q45. Give an example of a ring having identity but a subring of this having a different identity. **[2015]**

Q46. If R is a ring with unit element 1 and $\emptyset$ is a homomorphism of R onto R', prove that $\emptyset(1)$ is the unit element of R'. **[2015]**

Q47.Do the following sets form integral domains with respect to ordinary addition and multiplication? If so, state if they are fields:

(i) The set of numbers of the form $b\sqrt{2}$ with b ration
(ii) The set of even integers
(iii) The set of positive integers **[2015]**

Q48. Let k be a field and $k[x]$ be the ring of polynomial over k in a single variable x. For a polynomial $f \in k[x]$ generated by f. Show that (f) is maximal ideal in $k[x]$ if & only if f is an irreducible polynomial over k. **[2016]**

Q49. Let k be an extension of a field F. Prove that the element of k, which are algebraic over F, form a subfield of k. Further, if F C K C L are fields, L is algebraic over K and K is algebraic over F, then prove that L is algebraic over F **[2016]**

Q50. Show that every algebraically closed field is infinite. **[2016]**

Q51. Let F be a field and $F[x]$ denote the ring of polynomials over F in a single variable x. For $f(x), g(x) \in f(x)$ with $g(x) \neq 0$, show that there exist $q(x), r(x) \in f(x)$ such that degree $(r(x))$<degree $(g(x))$ and $f(x) = q(x)g(x) + r(x)$ **[2017]**

Q52. Let R be an integral domain with unit element. Show that any unit in $R[x]$ is a unit in R. **[2018]**

Q53. Find all proper subgroups of the multiplicative group of the field $(Z_{13}, +_{13}, X_{13})$ where $+_{13}$ and X_{13} represent additive module 13 and multiplication module 13 respectively. **[2018]**

Q54. Let a be a irreducible element of the Euclidean ring R, then prove that $R \mid (a)$ is a field. **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Real Analysis

TUTORIAL SHEET 4: Neighborhood, singularity and Real number system

Q1. Given a positive real number a and any natural number n, prove that there exists one and only one positive real number such that ε

$\varepsilon^n = a$ **[2007]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 5: Sequence & Series

Q1. Rearrange the series $\sum_{n=1}^{\infty}(-1)^{n+1}$ to converge to 1. **[2007]**

Q2. Discuss the convergence of the series $\frac{x}{2}+\frac{1\cdot 3}{2\cdot 4}x^2+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+....,\ x>0$ **[2008]**

Q3. Show that series $\sum\frac{1}{n(n+1)}$ is equivalent to $\frac{1}{2}\prod_{2}^{\infty}\left(1+\frac{1}{n^2-1}\right)$ **[2008]**

Q4. Show that the series

$\left(\frac{1}{3}\right)^2+\left(\frac{1.4}{3.6}\right)^2+\cdots+\left(\frac{1.4.7....(3n-2)}{3.6.9......3n}\right)^2+\cdots$ converges. **[2009]**

Q5. Discuss the convergence of the sequence $\{x_n\}$ where $x_n=\frac{\sin\left(\frac{n\pi}{2}\right)}{8}$. **[2010]**

Q6. Define $\{x_n\}$ by $x_1=5$ and $x_{n+1}=\sqrt{4+x_n}$ for $n>1$. Show that the sequence converges to $\left(\frac{1+\sqrt{17}}{2}\right)$.

[2010]

Q7. Consider the series

$\sum_{n=0}^{\infty}\frac{x^2}{(1+x^2)^n}$

Find the value of x for which it is convergent and also the sum function. Is the converge uniform? Justify your answer. **[2010]**

Q8. Show that the series for which sum f first n terms $f_n(x)=\frac{nx}{1+n^2x^2}$, $0\le x\le 1$ can not be differentiated terms at $x=0$. What happens at $x\neq 0$? **[2011]**

Q9. Show that if $S(x)=\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$, then its derivative $S'(x)=-2x\sum_{n=1}^{\infty}\frac{1}{n^2(1+nx^2)^2}$ for all x. **[2011]**

Q10. Show that the series $\sum_{n=1}^{\infty}\left(\frac{\pi}{\pi+1}\right)^n n^6$ is convergent. **[2012]**

Q11. Show that the series $\sum_{1}^{\infty}\frac{(-1)^{n-1}}{n+x^2}$ is uniformly convergent but not absolutely for all real values of x. **[2013]**

Q12. Test the convergence and absolute convergence of series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$ **[2015]**

Q13. Two sequences $\{x_n\}$ and $\{y_n\}$ are defined inductively by the following:

$x_1 = \frac{1}{2},\ y_1 = 1$ and $x_n = \sqrt{x_{n-1}y_{n-1}}$

$n = 2, 3, 4, \ldots$

$\frac{1}{y_n} = \frac{1}{2}\left(\frac{1}{x_n} + \frac{1}{y_{n-1}}\right),\ n = 2, 3, 4, \ldots$

Prove that $x_{n-1} < x_n < y_{n-1},\quad n = 2, 3, 4, \ldots$ and deduces the both the sequence converge to same limit l, where $\frac{l}{2} < l < 1$. **[2016]**

Q14. Show that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n+1}$ is conditionally convergent. If you use any theorem(s) to show it, then you must give a proof of that theorem(s). **[2016]**

Q15. Let $x_1 = 2$ and $x_{n+1} = \sqrt{x_n + 20},\ n = 1, 2, 3, \ldots.$ Show that the sequence $x_1, x_2, x_3, \ldots.$ is convergent. **[2017]**

Q16. Let $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series of real number. Show that there is a rearrangement

$$\sum_{n=1}^{\infty} x_{\pi}(n)$$

of series $\sum_{n=1}^{\infty} x_n$ that converges to 100. **[2017]**

Q17. Find the range of $p(>0)$ for which the series $\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \ldots.,\ a > 0$ is

(i) absolutely convergent (ii) conditionally convergent

[2018]

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 6: Limit, Continuity & differentiability

Q1. Prove that function f defined by

$$f(x)=\begin{cases}1 & \text{when } x \text{ is rational}\\ -1 & \text{when } x \text{ irrational}\end{cases}$$

is nowhere continuous. **[2006]**

Q2. A twice differentiable function f is such that $f(a)=f(b)=0$ and $f(c)>0$ for $a<c<b$. Prove that there is at least are value ξ, $a<\xi<b$ for which $f''(\xi)<0$

[2006]

Q3. Show that the function given by $f(x,y)$

$$=\begin{cases}\dfrac{x^3+2y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$$

continuous at $(0,0)$ (ii) posses partial derivative $f_x(0,0)$ and $f_y(0,0)$. **[2006]**

Q4. Show that function given by $f(x,y)=\begin{cases}\dfrac{xy}{x^2+2y^2}, & (x,y)\neq 0\\ 0 & ,(x,y)=0\end{cases}$ is not continuous at $(0,0)$ but its partial derivative f_x and f_y exist at $(0,0)$.

[2007]

Q5. If $f:R\to R$ is continuous and $f(x+y)=f(x)+f(y)$ for all $x,y\in R$ then show that $f(x)=x\,f(1)$ for all $x\in R$ **[2008]**

Q6. Let $f(x)=\begin{cases}\dfrac{|x|}{2}+1, & \text{if } x<1\\ \dfrac{x}{2}+1, & \text{if } 1\le x<2,\\ \dfrac{-|x|}{2}+1, & \text{if } 2\le x\end{cases}$

What are the point of discontinuity of f, if any? What are the points where f is not differentiable, if any? Justify your answer. **[2009]**

Q7. Show that if $f:[a,b]\to R$ is a continuous function then $f([a,b])=[c,d]$, form some real numbers c and d, $c\le d$.

[2009]

Q8. Show that $\lim_{x\to 1}\sum_{n=1}^{\infty}\frac{n^2x^2}{n^4+x^4}=\sum_{n=1}^{\infty}\frac{n^2}{n^4+1}$. Justify all steps of your answer by quoting theorem you are using.

[2009]

Q9. Show that a bounded infinite subset R must have a limit point. **[2009]**

Q10. Define the function $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{if } x\ne 0 \\ & \text{if } x=0\end{cases}$. Find $f'(x)$. Is $f'(x)$ continuous at $x=0$? Justify your answer. **[2010]**

Q11. Let $S=(0,1)$ and f be defined by $f(x)=\frac{1}{x}$, where $0<x\le 1$ (in R). Is f uniformly continuous on S? Justify your answer.

[2011]

Q12. Let $f(x,y)=\begin{cases}\frac{(x+y)^2}{x^2+y^2}, & \text{if } (x,y)\ne(0,0) \\ 1 \quad , & \text{if } (x,y)=(0,0)\end{cases}$. Show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at $(0,0)$ though $f(x,y)$ is not continuous at $(0,0)$. **[2012]**

Q13. Let $f(x)$ be differentiable on $[0,1]$ such that $f(1)=f(0)=0$ and $\int_0^1 f^2(x)dx=1$. Prove that $\int_0^1 xf(x)f'(x)=\frac{-1}{2}$. **[2012]**

Q14.Obtain $\frac{\partial^2 f(0,0)}{\partial x\partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y\partial x}$ for the function $f(x,y)=\begin{cases}\frac{xy(3x^2-2y^2)}{x^2+y^2}, & (x,y)\ne(0,0) \\ 0 & ,(x,y)\ne(0,0)\end{cases}$. Also, discuss continuously $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at $(0,0)$. **[2014]**

Q15. For the function $f:(0,\infty)\to R$ given by $f(x)=x^2\sin\left(\frac{1}{x}\right)$, $0<x<\infty$. Show that there is a differentiable function $g=R\to R$ that extendable f. **[2016]**

Q16. Let $f(t)=\int_0^t [x]dx$, where $[x]$ denotes the largest integer less than or equal to x.

(i) Determine all real numbers t at which f is differentiable.

(ii) Determine all real numbers t at which f is continuous but not differentiable.

[2017]

Q17. Show that if a function f defined on an open interval (a,b) of R is convex, then f is continuous. Show, by example, if the condition of open interval is dropped, then the convex function need not be continuous. **[2018]**

Q18. Suppose R be the set of all real numbers and $f: R \to R$ is function such that the following equation holds for all $x, y \in R$

(i) $f(x+y)=f(x)+f(y)$

(ii) $f(xy)=f(x)f(y)$. Show that $\forall\ x \in R$ either $f(x)=0$ or $f(x)=x$. **[2018]**

Q19. Show that function $f(x,y)=\begin{cases} \dfrac{x^2-y^2}{x-y}, & (x,y)\neq(1,-1),(1,1) \\ 0, & (x,y)=(1,1),(-1,1) \end{cases}$ is continuous differentiable at $(1,-1)$.

[2019]

Q20. Using Differentials, find an approximate value of $f(4\cdot 1, 4\cdot 9)$, where

$$f(x,y)=\left(x^3+x^2y\right)^{1/2}$$ **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 7: Mean Value Theorem / Maxima Minima

Q1. If f' and g' exist for every $x \in [a,b]$ and if $g'(x)$ does not vanish anywhere (a,b), show that there exist c in (a,b) such that

$$\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f'(c)}{g'(c)}$$ **[2005]**

Q2. Using Lagrange's mean value theorem show that $|\cos b - \cos a| \le |b-a|$ **[2007]**

Q3. For $x>0$, show that $\frac{x}{1+x} < \log(1+x) < x$ **[2008]**

Q4. Let f be a continuous function on $[0,1]$. Using first mean value theorem on integration, prove that $\lim_{n\to\infty}\int_0^1 \frac{nf(x)}{1+n^2x^2}dx = \frac{\pi}{2}f(0)$ **[2008]**

Q5. Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}, x \in \left(0,\frac{\pi}{2}\right)$ **[2008]**

Q6. State Rolle's theorem. Use it to prove that between two roots of $e^x \cos x = 1$ there will be a root $e^x \sin x = 1$ **[2009]**

Q7. Find the shortest distance from the origin $(0,0)$ to the hyperbola

$x^2 + 8xy + 7y^2 = 225$ **[2011]**

Q8. Find minimum distance of the line given by the plane $3x+4y+5z=7$ and $x-z=9$ and from the origin, by the method of Lagrange's multiplier

[2012]

Q9. Let $f(x,y) = y^2 + 4xy + 3x^2 + x^3 + 1$. What point will $f(x,y)$ have a maximum or minimum? **[2013]**

Q10. Find minimum value of $x^2+y^2+z^2$ subject to condition $xyz = a^3$ by method of Lagrange multiplier **[2014, 2015]**

Q11. Find absolute maximum and minimum value of $f(x,y) = x^2 + 3y^2 - y$ over the region

$x^2 + 2y^2 \le 1$ **[2015]**

Q12. Find the relative maximum and minimum values of function

$$f(x,y)=x^4+y^4-2x^2+4xy-2y^2$$ **[2016]**

Q13. Find the supremum and infimum of $\frac{x}{\sin x}$ on the interval $\left(0,\frac{\pi}{2}\right)$ **[2017]**

Q14. Prove the inequality $\frac{\pi^2}{9}<\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{x}{\sin x}dx<\frac{2\pi^2}{9}$ **[2018]**

Q15. Find maximum value of $f(x,y,z)=x^2y^2z^2$ subject to subsidiary condition

$x^2+y^2+z^2=c^2\,(x,y,z>0)$ **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 8: Riemann Integrals, Multiple Integral

Q1. Evaluate $\iiint \ln(x+y+z)dxdydz$ the integral being extended over all positive values of x, y, z such that $x+y+z \le 1$.

[2005]

Q2. Find the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **[2006]**

Q3. Find the volume of solid in the first octant bounded by paraboloid $z = 36-4x^2-9y^2$.

[2007]

Q4. Give an example of a function $f(x)$ that not Reimann integrable but $|f(x)|$ is Reimann integrable. Justify your answer. **[2012]**

Q5. Let $f(x)=\begin{cases} \frac{x^2}{2}+4 & \text{if } x \ge 0 \\ \frac{-x^2}{2}+2 & \text{if } x<0 \end{cases}$. Is f Reimann integrable in the interval $[-1,2]$? Why? Does there exist a function g such that $g'(x)=f(x)$? Justify your answer.

[2013]

Q6. Let $[x]$ denote integer part of real number x i.e. if $n \le x < n+1$, where n is an integer, then $[x]=n$. Is the function $f(x)=[x]+3$ Reimann integrable in the function $[-1,2]$? If not, explain why, if it is integrable, compute $\int_0^1 ([x]^2+3)dx$.

[2013]

Q7. Integrate $\int_0^1 f(x)dx$, where

$$f(x)=\begin{cases} 2x\sin\frac{1}{x}-\cos\frac{1}{x}, & x \in [0,1] \\ 0 \quad , & x=0 \end{cases}$$

[2014]

Q8. Is the function $f(x)=\begin{cases} \frac{1}{x}, & \frac{1}{n+1} < x \le \frac{1}{n} \\ 0, & x=0 \end{cases}$ Reimann integrable? If yes, obtain value of $\int_0^1 f(x)dx$. **[2015]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 9: Uniform Convergence

Q1. Let $f_n(x) = x^n \quad -1 < x \le 1$ for $n = 1,2,....$ Find the limit function. Is the convergence uniform? Justify your answer. **[2010]**

Q2. Let $f_n(x) = nx(1-x)^n, \quad x \in [0,1]$. Examine the uniform convergence of $\{f_n(x)\}$ on $[0,1]$. **[2011]**

Q3. Let $f_n(x) = \begin{cases} 0 & , \text{ if } x < \dfrac{1}{n+1} \\ \sin\dfrac{\pi}{x}, & \text{if } x < \dfrac{1}{n+1} \le x \le \dfrac{1}{n} \\ 0 & , \text{ if } x > \dfrac{1}{n} \end{cases}$, show that $f_n(x)$ converges to a continuous function but not uniformly. **[2012]**

Q4. Show that the series $\sum_{1}^{\infty} \dfrac{(-1)^{n-1}}{n+x^2}$ is uniformly convergent, but not absolutely for all real value of x. **[2013]**

Q5. Test series of functions $\sum_{n=1}^{\infty} \dfrac{nx}{1+n^2x^2}$ for uniform convergence. **[2013]**

Q6. Let $f : R \to R$ be a continuous function such that $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ erest and are finite. Prove that f is uniformly continuous on R. **[2016]**

Q7. Discuss the uniform convergence of $f_n(x) = \dfrac{nx}{1+n^2x^2} \ \forall x \in R(-\infty,\infty), \ n = 1,2,3,....$ **[2019]**

Q8. Discuss the convergence of $\int_1^2 \dfrac{\sqrt{x}}{\ln x} dx$. **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 10: Improper Integrals

Q1. Show that $\int_0^\infty e^{-t}t^{n-1}dt$ is an improper integer which converges for $n>0$. **[2005]**

Q2. Examine the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$. **[2005]**

Q3. Test the convergence of the improper integral $\int_1^\infty \frac{dx}{x^2(1+e^{-x})}$. **[2014]**

CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Complex Analysis

TUTORIAL SHEET 11: Analytic Functions, Cauchy Riemann Equations

Q1. If $f(z)=u+iv$ is an analytic function of the complex variable z and

$$u-v=e^{x}(\cos y-\sin y),$$

determine $f(z)$ in terms of z. **[2005]**

Q2. Prove that the function f defined by

$$f(z)=\begin{cases}\dfrac{z^5}{|z|^4}, & z\neq 0\\ 0, & z=0\end{cases}$$

is not differentiable at $z=0$. **[2007]**

Q3. Show that $u(x,y)=2x-x^3+3xy^2$ is a harmonic function. Find a harmonic conjugate of $u(x,y)$. Hence find the analytic function f for which $u(x,y)$ is the real part.

[2010]

Q4. If $f(z)=u+iv$ is an analytic function of $z=x+iy$ and $u-v=\dfrac{e^y-\cos x+\sin x}{\cosh y-\cos x}$,

find $f(z)$ subject to condition, $f\left(\dfrac{\pi}{2}\right)=\dfrac{3-i}{2}$.

[2011]

Q5. Show that the function defined by

$$f(z)=\begin{cases}\dfrac{x^3y^5(x+iy)}{x^6+y^{10}}, & z\neq 0\\ 0, & z=0\end{cases}$$

is analytic at the origin through it satisfies Cauchy Riemann equation at the origin.

[2012]

Q6. Prove that function $f(z) = u + iv$, where $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$, $f(0) = 0$ satisfies Cauchy Riemann equation at the origin, but the derivative of f at $z = 0$ does not exist. **[2014]**

Q7. Show that the function $v(x,y) = \ln(x^2 + y^2) + x + y$ is harmonic. Find its conjugate harmonic function $u(x,y)$. Also, find the corresponding analytic function $f(z) = u + iv$ in term of z. **[2015]**

Q8. Let $f = u + iv$ be an analytic function in unit $D = \{z \in C : |z| < 1\}$. Show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0 = \dfrac{\partial^2 V}{\partial x^2} + \dfrac{\partial^2 V}{\partial y^2}$ at all points of D. **[2017]**

Q9. For a function $f: C \to C$ and n ≥ 1, let f^n denotes the n^{th} derivative of f and $f^0 = f$. Let f be an entire function such that for some $n \geq 1$, $f^n\left(\frac{1}{k}\right) = 0$for all $k = 1, 2, 3, ...$ Show that f is a polynomial. **[2017]**

Q10. Prove that function $u(x,y) = (x-1)^3 - 3xy^2 + 3y^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z. **[2018]**

Q11. Suppose $f(z)$ is analytic function on a domain D in c and satisfy the equation $Im\, f(z) = (Re\, f(z))^2\ Z \in D$ show that $f(z)$ is constant in D **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 12: Power Series, Taylor's Series and Laurent's Series

Q1. Expand $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent's series which is valid for

(i) $1<|z|<3$, (ii) $|z|<3$ (iii) $|z|<1$

Q2. Find the Laurent series of the function $f(z)=\exp\left[\dfrac{\lambda}{2}\left(z-\dfrac{1}{2}\right)\right]$ as $\sum_{n=-\infty}^{\infty} C_n z^n$ for 0, $|z|<\infty$, where $C_n=\int_0^{\pi}\cos(n\phi-\lambda\sin\phi)\,d\phi$, $n=0,\pm1,\pm2,....$ with λ a given complex number and taking the unit circle C given by $z=e^{i\phi}\,(-\pi\le\phi\le\pi)$ as contour in this region.

[2010]

Q3. Find the Laurent series for the function $f(z)=\dfrac{1}{1-z^2}$ with centre $z=1$ **[2011]**

Q4. Expand the function $f(z)=\dfrac{1}{(z+1)(z+3)}$ in Laurent series valid for

(i) $1<|z|<3$ (ii) $|z|<3$ (iii) $0<|z+1|<2$ (iv) $|z|<1$ **[2012]**

Q5. Prove that if $be^{a+1}<1$, where a and b are positive and real, then the function $z^n e^{-a}-be^{z}$ has n zeroes in the unit circle.

[2013]

Q6. Expand in Laurent series the function $f(z)=\dfrac{1}{z^2(z-1)}$ about $z=0$ and $z=1$.

[2014]

Q7. Find all possible Taylor's and Laurent's series expansion of function $f(z)=\dfrac{2z-3}{z^2-3z+2}$ about the point $z=0$.

[2015]

Q8. Prove that every even power series represents an analytic function inside its circle of convergence. **[2016]**

Q9. Find the Laurent series which represent the function $\frac{1}{(1+z^2)(z+2)}$ when

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$ **[2018]**

Q10. Obtain the first three terms of the Laurent series expansion of the function $f(z) = \frac{1}{(e^z-1)}$ about the point $z = 0$ valid in the reason $0 < |z| < 2\pi$

[2019]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 13: Complex Integration, Cauchy's theorem, Cauchy Integral formula

Q1. If α, β, γ are real numbers such that $\alpha^2 > \beta^2 + \gamma^2$, show that:

$$\int_0^{2\pi} \frac{d\theta}{\alpha + \beta\cos\theta + \gamma\sin\theta} = \frac{2\pi}{\sqrt{\alpha^2 - \beta^2 - \gamma^2}}$$ **[2009]**

Q2. Evaluate the line integral $\int_C f(z)\,dz$ where $f(z) = z^2, C$ is the boundary of the triangle with values $(0,0), B(1,0), (1,2)$, in that order. **[2010]**

Q3. If the function $f(z)$ is analytic and one value is $|z-a| < R$, prove that $0 < r < R$,

$$f'(a) = \frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}\,d\theta$$, where $P(\theta)$ is the real part of $f\left(a + re^{i\theta}\right)$ **[2011]**

Q4. Use Cauchy integral formula to evaluate $\int_c \frac{e^{3z}}{(1+z)^4}\,dz$, where C is the

curve $|z| = 2$

[2012]

Q5. Evaluate the integral $\int_C \mathrm{Re}\left(z^2\right)dz$ from 0 to $2+4i$ along the curve C,

where C is a parabola $y = x^2$.

[2019]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 14: Cauchy's Residue theorem, singularities, Contour integration

Q1. With the aid of residues, evaluate $\int_0^{\pi} \frac{Cos2\theta}{1-2acos\theta+a^2} d\theta \quad -1<a<1$ **[2006]**

Q2. Evaluate (by using residue theorem) $\int_0^{2\pi} \frac{d\theta}{1+8cos^2\theta}$ **[2007]**

Q3. Evaluate $\int_c \left[\frac{e^{2z}}{z^2(z^2+2z+2)} + \log(z-6) + \frac{1}{(z-4)^2}\right] dz$ where C is the circle $|z|=3$. State the theorem you use in evaluating above integral.

[2008]

Q4. Find the residue of $\frac{\cot z \cot hz}{z^3}$ at $z=0$. **[2008]**

Q5. Let $f(z)=\dfrac{a_0+a_1z+....+a_{n-1}z^{n-1}}{b_0+b_1z+....b_nz^n}$, $b_n \neq 0$. Assume that the zeroes of the denomination are simple. Show that the sum of the residues of $f(z)$ at its poles is equal to $\dfrac{a_n-1}{b_n}$.

[2009]

Q6. Evaluate by Contour integration

$$\int_0^1 \frac{dx}{(x^2-x^3)^{1/3}}$$

[2011]

Q7. Evaluate by Contour integration

$$I = \int_0^{2\pi} \frac{d\theta}{1-2a\cos\theta+a^2}, a^2<1$$ **[2012]**

Q8. Using Cauchy's residue theorem, evaluate the integral

$$I = \int_0^{\pi} \sin^4\theta d\theta$$ **[2013]**

Q9. Evaluate $\int_0^{\pi} \dfrac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$ using residue. **[2014]**

Q10. State Cauchy's residue theorem. Using it, evaluate

$$\int_c \frac{e^z+1}{z(z+1)(z-i)^2} dz, C: |z|=2$$ **[2015]**

Q11. Let $\gamma:[0,1]\to C$ be the curve $\gamma(t)=e^{2\pi i t},\ 0\le t\le 1$ find giving justification the value the Contour integration.

$\int_\gamma \frac{dz}{4z^2-1}$ **[2016]**

Q12. Determine all entire function $f(z)$ such that 0 is a removable singularity of $f\left(\frac{1}{z}\right)$.

[2017]

Q13. Using Contour integral method, prove

$$\int_0^\infty \frac{x\sin mx}{x^2+a^2}dx=\frac{\pi}{2}e^{--ma}$$ **[2017]**

Q14. Show by applying the residue theorem that

$$\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^2}=\frac{\pi}{4a^3},\ a>0$$ **[2018]**

Q15.Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x\left(1+x^2\right)}dx,\ a>0, a\ne 1$ **[2019]**

Q16. Show that an isolated singular point z_0 of a function $f(z)$ is a pole of order m if and only if $f(z)$ can be written in the form $f(z)=\frac{\phi(z)}{(z-z_0)^m}$, where $\phi(z)$ is analytic and non-zero at z_0. Moreover, $\operatorname*{Res}_{z=z_0} f(z)=\frac{\phi^{(m-1)}(z_0)}{(m-1)!}$, if $m\ge 1$. **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)
Mathematics Paper II: Linear Programming Problems
TUTORIAL SHEET 15: Linear Programming Problems

Q1. Put the following program in standard form

Minimize $z = 25x_1 + 30x_2$

Subject to $4x_1 + 7x_2 \geq 1$

$8x_1 + 5x_2 \geq 3$

$6x_1 + 9x_2 \geq -2$

and hence obtain an initial feasible solution. **[2005]**

Q2. Use simplex method to solve the following

Maximize $z = 5x_1 + 2x_2$

Subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0$ **[2005]**

Q3. Given the programme

$u = 5x + 2y$

$x + 3y \leq 12$

$3x - 4y \leq 9$

$7x + 8y \leq 20$

$x, y \leq 0$

Write the dual in standard form. **[2006]**

Q4. Use the simpler method to solve

Maximize $u = 2x + 3y$

Subject to $-2x + 3y \leq 2$

$3x + 2y \leq 5$

$x, y \geq 0$ **[2006]**

Q5. Put the following in slack form and describe which of the variables are 0 at each of the vertices of the constraint set and hence determine the vertices algebraically

Maximize $u = 4x + 3y$

Subject to $x + y \le 4$

$-x + y \le 2$

$x, y \ge 0$ **[2007]**

Q6. Solve the following by simplex method:

Maximize $u = 4x + 3y$

Subject to $x + y \le 1$

$-x - 2y \le 4$

$x, y \ge 0$ **[2007]**

Q7. Find the dual of the following L.P.P.

Maximize $z = 2x_1 - x_2 + x_3$

Such that $x_1 + x_2 - 3x_3 \le 8$

$4x_1 - x_2 + x_3 = 2$

$2x_1 + 3x_2 - x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$ **[2008]**

Q8. Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \le 8$

$3x_1 + 2x_2 + 4x_3 \le 15$

$2x_2 + 5x_3 \le 10$

$x_1 \ge 0$ **[2009]**

Q9. Construct the dual of the primal problem:

Maximize $z = 2x_1 + x_2 + x_{3i}$

Subject to constraint $x_1 + x_2 + x_3 \ge 6,\ 3x_1 - 2x_2 + 3x_3 = 3$

$-4x_1 + 3x_2 - 6x_3 = 1$ and $x_1, x_2, x_3 \ge 0$ **[2010]**

Q10. Solve the following L.P. Problem by simplex method:

Maximize $z = 5x_1 + 3x_2$

Constraint $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$ **[2011]**

Q11. Write down the dual of the following L.P.P. problem and hence solve it by graphical method.

Maximize $z = 6x_1 + 4x_2$

Constraint $2x_1 + x_2 \le 1$

$3x_1 + 4x_2 \ge 1.5$

$x_1, x_2 \ge 0$ **[2011]**

Q12. For each hour per day that Ashok studies Mathematics it yields him 10 marks and for each hour that he studies Physics it yields him 5 marks he can study at most 14 hours a day and he must get atleast 40 marks in each. Draw graphically how many hours a day he should study Mathematics and Physics each in order to maximize his marks?

[2012]

Q13. Maximize $z = 2x_1 + 3x_2 - 5x_3$

Subject to $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \ge 10$, $x_i \ge 0$ **[2013]**

Q14. Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

Subject to constraint $x_1 + 2x_2 - 2x_3 + 4x_4 \le 40$

$2x_1 - x_2 + x_3 + 2x_4 \le 8$

$4x_1 - 2x_2 + x_3 - x_4 \le 10$, $x_i \ge 0$ **[2013]**

Q15. Solve graphically

Maximize $z = 6x_1 + 5x_2$

Subject to $2x_1 + x_2 \le 16$

$x_1 + x_2 \le 11$

$x_1 + 2x_2 \ge 6$

$5x_1 + 6x_2 \le 90$, $x_1, x_2 \ge 0$ **[2014]**

Q16. Find all the optimal solution of the following L.P.P. by simplex method:

Maximize $z = 30x_1 + 24x_2$

Subject to $5x_1 + 4x_2 \le 200$

$x_1 \le 32$

$x_2 \le 40 \qquad x_1, x_2 \ge 0$ **[2014]**

Q17. Consider the L.P.P.

Maximize $z = x_1 + 2x_2 - 3x_3 + 4x_4$

Subject to $x_1 + x_2 + 2x_3 + 3x_4 = 12$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \ge 0$

(i) Using the definition, find its all basic solutions which of these are degenerate basic feasible solution and which are non degenerate basic feasible solutions.

(ii) Without solving the problem show that it has an optimal solution and which of the basic feasible solution is/are optimal. **[2015]**

Q18. Solve the following L.P.P. using simplex method, write its dual. Also, write the optimal table of the given problem

Maximize $z = 2x, -4x_2 + 5x_3$

Subject to $x_1 4x_2 - 2x_3 \ge 2$

$-x_1 + 2x_2 + 3x_3 \le 1$

$x_1, x_2, x_3 \ge 0$ **[2015]**

Q19. Find the maximum value of $5x + 2y$ with constraint $x + 2y \ge 1,\ 2x + y \le 1,\ x > 0,\ y \ge 0$ by graphical method.

[2016]

Q20. Maximize $z = 2x_1 + 3x_2 + 6x_3$

Subject to $2x_1 + x_2 + x_3 \le 5$

$3x_2 + 2x_3 \le 6$

$x_1 \ge 0, x \ge 0, x_3 \ge 0$

is the optimal solution unique? Justify your answer. **[2016]**

Q21. Using graphical method, find maximum value of $2x+y$

Subject to $4x+3y\le 12$

$4x+y\le 8$

$4x-y\le 8, x, y\ge 0$ **[2017]**

Q22. Solve the L.P.P. by simplex method,

Maximize $z=3x_1+5x_2+4x_3$

Subject to $2x_1+3x_2\le 8$

$2x_2+5x_3\le 10$

$3x_1+2x_2+4x_3\le 15$

$x_1, x_2, x_3\ge 0$ **[2017]**

Q23. An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphates and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3 : 3 : 4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2 : 4 : 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. **[2018]**

Q24. Solve the following L.P.P. by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function

Minimize $z=3x_1+5x_2$

Subject to $x_1+2x_2\ge 8$

$3x_1+2x_2\ge 12$

$5x_1+6x_2\le 60$

$x_1, x_2\ge 0$ **[2018]**

Q25. How many basic solutions are there in the following linearly independent set of equations? Find all of them

$2x_1-x_2+3x_3+x_4=6$

$4x_1-2x_2-x_3+2x_4=10$ **[2018]**

Q26. Use graphical method to solve L.P.P.

Maximize $z = 3x_1 + 2x_2$

$x_1 - x_2 \geq 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$ **[2019]**

Q27.) Solve the L.P.P. using simplex method .

Minimize $z = x_1 + 2x_2 - 3x_3 - 2x_4$

$x_1 + 2x_2 - 3x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + 2x_4 = 4$

and $x_1, x_2, x_3, x_4 \geq 0$

[2019]

Q28. Consider the L.P.P.

Maximize : $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

$x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8 \qquad x_1, x_2, x_3, x_4 \geq 0$

Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal

[2019]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET -16: Transport Problems

Q1. Solve the following transportation problem

	D_1	D_2	D_3	D_4	D_5	D_6	Available
F_1	2	1	3	3	2	5	50
F_2	3	2	2	4	3	4	40
F_3	3	5	4	2	4	1	60
F_4	4	2	2	1	2	2	30
Demand	30	50	20	40	30	10	

By finding the initial solution by matrix minima method. **[2008]**

Q2. Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories F_1, F_2, F_3 to fuse warehouses w_1, w_2, w_3, w_4, w_5 is minimized. The fuse warehouse must received 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The transportation cost per ton from factories to warehouse are given in table below:

	w_1	w_2	w_3	w_4	w_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution **[2010]**

Q3. By the method of Vogel, determine an initial basic solution for the following transportation problem:

Products P_1, P_2, P_3 & P_4 have to be sent to destination D_1, D_2, D_3. Cost of sending product P_i to destinations Pj or $C_i j$, where the matrix

$$[C_i j] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}$$

Total requirement of destination D_1, D_2 and D_3 are 15, 45, 95 respectively and availability of product P_1, P_2, P_3 and P_4 are respectively 25, 35, 55 and 70. **[2012]**

Q5.

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	

Find the initial feasible solution to the following transportation problem by Vogel's approximation method. Also, find the optimal solution and the minimum transport cost.

[2014]

Q6. Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost

	D_1	D_2	D_3	D_4	D_5	
O_1	4	7	0	3	6	14
O_2	1	2	- 3	3	8	9
O_3	3	- 1	4	0	5	17
	8	3	8	13	8	

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 17: Assignment Problem

Q1. Solve the minimize time assignment problem

J_1	3	12	5	14
J_2	7	9	8	12
J_3	5	11	10	12
J_4	6	14	4	11

[2013]

Q2. Solve the following assignment problem to maximize the sales:

	I	II	III	IV	V
A	3	4	5	6	7
B	4	15	13	7	6
C	6	13	12	5	11
D	7	12	15	8	5
E	8	13	10	6	9

[2015]

Q3.

	M_1	M_2	M_3	M_4	M_5
O_1	24	29	18	32	19
O_2	17	26	34	22	21
O_3	27	16	28	17	25
O_4	22	18	28	30	24
O_5	28	16	31	24	27

In a factory, there are 5 operators O_1, O_2, O_3, O_4 and O_5 and 5 machines M_1, M_2, M_3, M_4 and M_5. The operating cost is given when the O_1 operator operates the M_{ij} machines $(i, j = 1, 2, ..., 5)$ but there is restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate on 5th machine M_5. Cost matrix is given above. Find optimal assignment cost also.

[2018]

CIVIL SERVICE EXAMINIATION (MAINS)

Section B

Mathematics Paper II: Partial Differential Equation

TUTORIAL SHEET 18: Framing of P D E & linear P D E of Order one

Q1. Find the integral surface of the following P.D.E. $x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$.

[2004]

Q2. Formulate P.D.E. for surface whose tangent planes form a tetrahedron of constant volume with coordinate planes. **[2005]**

Q3. Find the particular integral of $x(y-z)p+y(z-x)q=z(x-y)$. **[2005]**

Q4. Solve: $px\left(z-2y^2\right)=(z-qy)\left(z-y^2-2x^3\right)$ **[2006]**

Q5. Form a P.D.E. by eliminating the function f from $z=y^2+2f\left(\frac{1}{x}+\log y\right)$.

[2007]

Q6. Solve $2zx-px^2-2qxy+pq=0$. **[2007]**

Q7. Find the general solution of P.D.E. $(2xy-1)p+\left(z-2x^2\right)q=2(x-yz)$ and also find the particular solution which passes through the lines $x=1,\ y=0$. **[2008]**

Q8. Form the P.D.E. by eliminating the arbitrary function f given by $f\left(x^2+y^2,\ z-xy\right)=0$.

[2009]

Q9. Show that D.E. of all cones which have their vertex at the origin is $px+qy=z$. Verify that this equation is satisfied by the surface $yz+zx+qy=0$. **[2009]**

Q10. Solve the P.D.E. $(x+2z)p+(4xz-y)q=2x^2+y$. **[2011]**

Q11. Solve the P.D.E. $px+qy=3z$. **[2012]**

Q12. Form a P.D.E. by eliminating the arbitrary functions f and g from $z=y\,f(x)+x\,g(y)$

[2013]

Q13. Solve the partial differential equation
$(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ where
$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ **[2015]**

Q14. Solve for general solution
$p\ cos\ (x + y) + q\ sin\ (x + y) = z$ where
$p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ **[2015]**

Q15. Find the general equation of surfaces orthogonal to the family of spheres given by
$x^2 + y^2 + z^2 = Cz$ **[2016]**

Q16. Find the general integral of the P D E
$(y + zx)p - (x + yz)q = x^2 - y^2$ **[2016]**

Q17. Find the P D E of family of all tangent planes to the ellipsoid $x^2 + 4y^2 + 4z^2 = 4$ which are not perpendicular to my plane. **[2018]**

Q18. Find the general solution of P D E $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ & find its integral surface that passes through the curve $x = t, y = t^2, z = 1$ **[2018]**

Q19. Form a p d e of family of surfaces given by following expression
$\psi(x^2 + y^2 + 2z^2, y^2 - 2zx) = 0$ **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 19: P D E of degree more than one

Q1. Using Charpit method, find the complete internal of $2xz - px^2 - 2qxy + pq = 0$.

[1993]

Q2. Use Charpit's method to solve $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$. Interpret geometrically the complete solution and mention the singular solution.

[1994]

Q3. Explain in detail the Charpit's method of solving the nonlinear P.D.E. **[1995]**

Q4. Solve $z^2\left(p^2 + q^2 + 1\right) = c^2$. **[1996]**

Q5. Solve by Charpit's method: $z = px + qy + p^2 + q^2$. **[1996]**

Q6. Solve by Charpit's method: $z^2\left(p^2z^2 + q^2\right) = 1$. **[1997]**

Q7. Use Charpit's method to find complete integral of $\left[2x\left(z\dfrac{\partial z}{\partial y}\right)^2 + 1\right] = z\dfrac{\partial z}{\partial x}$.

[1998]

Q8. Use Charpit's method to find complete integral of $p^2 + q^2 - 2px - 2qy + 1 = 9$.

[1999]

Q9. Solve by Charpit's method $p^2x\left(x-1\right) + 2pq\,xy + q^2y\left(y-1\right) - 2pxz - 2qyz + z^2 = 0$

.

[2000]

Q10. Solve $2p^2q^2 + 3x^2y^2 = 8x^2q^2\left(x^2 + y^2\right)$. **[2001]**

Q11. Solve the equation $p^2 - q^2 - 2px - 2qy + 2xy = 0$ using Charpit's method also find the singular solution of the equation if it exists. **[2003]**

Q12. Using Charpit's method, find the complete solution of the P.D.E. $p^2x + q^2y = z$.

[2004]

Q13. Solve by Charpit's method $p^2x + q^2y = z$. **[2006]**

Q14. Find the complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method. **[2008]**

Q15. Find a complete integral of P D E
$2(pq + yp + qx) + x^2 + y^2 = 0$ **[2017]**

TUTORIAL SHEET 20: Linear P D E of 2nd order with constant coefficient

Q1. Find the general solution of

$$\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y).$$ **[2003]**

Q2. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-3\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=xy+e^x+2y$. **[2003]**

Q3. Solve the P.D.E. $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$. **[2004]**

Q4. Obtain the general solution of $(D-3D-2)^2 z=2e^{2x}\sin(y+3x)$. **[2005]**

Q5. Solve: $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial x\partial y^2}=2\sin(3x+2y)$. **[2006]**

Q6. Find the general solution of $(D^2+DD'-6D'^2)z=y\cos x$. **[2008]**

Q7. Solve: $(D^2-DD'-2D'^2)z=(2x^2+xy-y^2)\sin xy-\cos xy$. **[2009]**

Q8. Solve the P.D.E. $(D^2-D')(D-2D')z=e^{2x+y}+xy$. **[2010]**

Q9. Find the surface satisfying the P.D.E. $(D^2-2DD'+D'^2)z=0$ and the conditions that $bz=y^2$ when $x=0$ and $az=x^2$ when $y=0$. **[2010]**

Q10. Solve the P.D.E. $(D^2-D'^2+D+3D'-2)z=e^{x-y}-x^2y$. **[2011]**

Q11. Solve the P.D.E. $(D-2D')(D-D')^2 z=e^{x+y}$. **[2012]**

Q12. Solve: $(D^2+DD'-6D'^2)z=x^2\sin(x+y)$. **[2013]**

Q13. Solve the Partial Differential Equation $\left[2D^2-5DD'+2D'^2\right]z=24(y-x)$.

[2014]

Q14. Solve $(D^2+DD'-2D'^2)u=e^{x+y}$, where $D=\frac{\partial}{\partial x}$ & $D'=\frac{\partial}{\partial y}$

[2015]

Q15. Solve the p d e

$\frac{\partial^3 z}{\partial x^3}-z\frac{\partial^3 z}{\partial x^2\partial y}-\frac{\partial^3 z}{\partial x\partial y^2}+2\frac{\partial^3 z}{\partial y^3}=e^{x+y}$ **[2016]**

Q16. Solve $(D^2-2DD'+D'^2)z=e^{x+2y}+x^3+Sin2x$ **[2017]**

Q17. Solve $(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$
Where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$ **[2018]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 21: Application of PDE (Vibrating string; Heat Equation & Laplace Equation)

Q1. Find the deflection $u(x,t)$ of a vibrating string, stretches between fixed points $(0,0)$ and $(3l,0)$, corresponding to zero initial velocity and following initial deflection

$$f(x)=\frac{hx}{l} \quad \text{where } 0\le x\le l$$
$$=\frac{h(3l-2x)}{l} \quad \text{where } l\le x\le 2l$$
$$=\frac{h(x-3l)}{l} \quad \text{where } 2l\le x\le 3l$$

[2003]

Q2. A uniform string of length l held tightly between $x=0$ and $x=l$ with no initial displacement, is struck at $x=a$, $0<a<l$ with velocity v_0. Find the displacement of the string at any time $t>0$.

[2004]

Q3. The ends A and B of a rod 20 cm long have the temperature at 30^0C and at 80^0C until steady state prevails. The temperature of the ends are changed to 40^0C and 60^0C respectively. Find the temperature distribute in the rod at time t.

[2005]

Q4. The deflection of a vibrating string of length l is governed by P.D.E. $u_{tt}=c^2u_{xx}$. The ends of the string are fixed at $x=0$ and l. The initial velocity is zero. The initial displacement is given by

$$u(x,0)=\frac{x}{l} \quad 0<x<\frac{l}{2}=\frac{l-x}{l} \; \frac{l}{2}<x<l$$

Find the deflection of the string at any instant of time. **[2006]**

Q5. Solve $u_{xx}+u_{yy}=0$ in D, where $D:\{(x,y):0<x<a,\ 0<y<b\}$ is a rectangle in a plane with the boundary conditions

$$u(x,0)=0 \qquad u(x,b)=0 \qquad 0\le x\le a$$
$$u(0,y)=g(y)ux(a,y)=h(y) \qquad 0\le y\le b.$$

[2007]

Q6. Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions $u(0,t) = 0 = u(l,t)$ for all t and $u(x,0) = f(x)$ for all x in $[0,l]$.

[2007]

Q7. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0,\ x = a,\ y = 0,\ y = b$. The edges $x = 0,\ x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at $100^0 C$. **[2008]**

Q8. A tightly stretched string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant and then released. Find the displacement at any point x of the string at time $t > 0$. **[2009]**

Q9. Solve the following heat equation:

$$u_t - u_{xx} = 0 \qquad 0 < x < 2; \qquad t > 0$$

$$u(0,t) = u(2,t) = 0; \qquad t > 0$$

$$u(x,0) = x(2 - x) \qquad 0 \le x \le 2$$

[2010]

Q10. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \begin{matrix} 0 < x \le a \\ 0 < y \le b \end{matrix}$

Satisfying the boundary conditions

$$u(0,y) = 0, \qquad u(x,0) = 0, \qquad u(x,b) = 0$$

$$\frac{\partial u}{\partial x}(a,y) = T \sin^3 \frac{\pi y}{a}$$

[2011]

Q11. Obtain temperature distribution $y(x,t)$ in a uniform bar of unit length whose one end is kept at $10^0 C$ and the other end is insulated. Also it is given that $y(x,0) = 1 - x \qquad 0 < x < 1$ **[2011]**

Q12. A string of length l is fixed at its ends. The string from the mid point is pulled up to a height K and then released from rest. Find the deflection $y(x,t)$ of vibrating string.

[2012]

Q13. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **[2012]**

Q14. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in equilibrium position. If it is set vibrating by giving ends point a velocity $\lambda(x)(l-x)$. Find the displacement of the string at any distance x from one end at any time t. **[2013]**

Q15. Find the deflection of a vibrating string (length $=\pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$).
Corresponding to zero initial velocity and initial deflection $f(x)=k(\sin x-\sin 2x)$.

[2014]

Q16. Solve: $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}; \quad 0<x<1,$

Given that

(i) $u(x,0)=0 \qquad 0\le x\le 1$

(ii) $\frac{\partial u}{\partial t}(x,0)=x^2; \qquad 0\le x\le 1$

(iii) $u(0,t)=u(1,t)$ for all t. **[2014]**

Q17. Find the solution of the initial boundary value problem
$u_t - u_{xx} + u = 0 \quad , \quad 0<x<l, \quad t>0$
$u(0,t) = u\,(u,t) = 0, \quad t \ge 0$
$u(x,0) = x(l-x), \quad 0<x<l$ **[2015]**

Q18. Find the temperature $u(x,t)$ in a bar of silver of length 10 cm and constant cross section of area 1 cm^2. Let density P = 10.6 g/cm^3 , thermal conductivity $k = 1.04\ cal/(cm\,\sec{}^\circ c)$ and specific heat $\sigma = 0.056\ cal/g^\circ c$. The bar is perfectly isolated laterally with ends kept at $0^\circ C$ and initial temperature $(x) = Sin(0.1\pi x)^\circ c$. Note that $u(x,t)$ follows the heat equation $= c^2 uxx$, where $c^2 = k/(\rho\sigma)$ **[2016]**

Q19. Given the one dimensional wave equation
$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} \ , \ t>0$
Where $c^2 = \frac{T}{m}$. T is the constant tension in the string and m is the mass per unit length of the string.

(i) Find the appropriate solution of the above wave equation

(ii) Find also the solution under the conditions
$y(0,t) = 0\,,\ y(l,t) = 0\ for\ all\ t$
And $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0 \quad , \quad y(x,0) \ = \ a\sin\frac{\pi x}{l}\ ,$
$0<x<l \quad , \quad a>0$ **[2017]**

Q20. A thin annulus occupies the region $0<a\le r\le b\,, \quad 0\le\theta\le 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0° while along the outer edge the temperature is held at $T = k\cos\frac{\theta}{2}$ where k is a constant. Determine the temperature distribution in the annulus. **[2018]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 22: Canonical form/Cauchy's P D E

Q1. Show that the differential equation of all cones which have their vertex at the origin are $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

[2003]

Q2.Find the integral surface of the following P.D.E. $x\left(y^2 + z\right)p - y\left(x^2 + z\right)q = \left(x^2 - y^2\right)z$.

[2004]

Q3. Find the complete integral of the P.D.E. $\left(p^2 + q^2\right)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$. **[2004]**

Q4. Find the surface passing through the parabolas $z = 0, y^2 = 4ax$ and $z = 1, y^2 = -4ax$ and satisfying the equation $x\dfrac{\partial^2 z}{\partial x^2} + 2\dfrac{\partial z}{\partial x} = 0$. **[2006]**

Q5. Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of resolution. **[2007]**

Q6. Reduce $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ to canonical form. **[2008, 2014]**

Q7. Find the characteristics of $y^2 r - x^2 t = 0$, where r and t have their usual meanings.

[2009]

Q8.Find the integral surface of $x^2 p + y^2 q + z^2 = 0$ which passes through the $xy = x + y,\ z = 1$. **[2009]**

Q9. Reduce the following equations to canonical form and fixed its general solution $xu_{xx} + 2xu_{xy} - u_x = 0$. **[2010]**

Q10. Solve the following P.D.E. by the method of characteristics:

$$zp + yq = x$$

$$x(\dot{s}) = s, \qquad y_0(s) = 1, \qquad z_0(s) = 2s.$$ **[2010]**

Q11. Find the surface satisfying $\dfrac{\partial^2 z}{\partial x^2} = 6x + 2$ and trouching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. **[2011]**

Q12.Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1), C$ is constant, orthogonally and which passes through the circle $x^2+y^2=1,\ z=1$. **[2013]**

Q13. Reduce the equation to canonical form when $x \neq y$

$$y\frac{\partial^2 z}{\partial x^2}+(x+y)\frac{\partial^2 z}{\partial x \partial y}+x\frac{\partial^2 z}{\partial y^2}.$$ **[2014]**

Q14. Reduce the second order partial differential equation

$x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial a}+y\frac{\partial u}{\partial y}=0$ canonical form. Hence, find its general solution. **[2015]**

Q15. Determine characteristic of equation $z=p^2-q^2$ & find integral surface which passes through parabola 4z≠ $x^2=0\,, y=0$ **[2016]**

Q16. Reduce the equation

$y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x \partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$

to canonical form and hence solve it **[2017]**

Q17. Solve the first order quasi linear partial differential equation by method of characteristics

$x\frac{\partial u}{\partial x}+(u-x-y)\frac{\partial u}{\partial y}=x+2y$ in

$x>0,\ -\infty<y<\infty$ with $u=1+y$ on $x=1$ **[2019]**

Q18. Reduce the following second order PDE to canonical form & find the general solution

$\frac{\partial^2 u}{\partial x^2}-2x\frac{\partial^2 u}{\partial x \partial y}+x^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial y}+12x$ **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Numerical Method & C P

TUTORIAL SHEET 23: Solution of Algebric & Transcendental equation

Q1. Find a root of the equation $x\sin x + \cos x = 0$ using Newton-Rapson Method.

[1988]

Q2. The polynomial $x^3 - x - 1$ has a root between 1 and 2. Using the Secant method, find this root correct to 3 significant figures. **[1989]**

Q3. Solve $x^2 - 5x + 3 = 0$ in the interval $[1,2]$ by Secant Method. **[1990]**

Q4. Using Regula-Falsi method, find the real root of the equation $x\log_{10} x - 1.2 = 0$, correct to 5 decimal places. (Ans. 2.74064844) **[1991]**

Q5. Compute to 4 decimal places by using Newton-Raphson method, the real root of

$$x^2 + 4\sin x = 0$$

[1992]

Q6. Find correct to 3 decimal places the real root of $2e^x - 3x^2 = 2.5644$. **[1993]**

Q7. Find the positive root of $\log_e x = \cos x$, nearest to 5 decimal places by Newto-Raphson method. **[1995]**

Q8. Describe Newton-Raphson method for finding the solutions of the equation $f(x) = 0$ and show that method has a quadratic convergence.

[1996]

Q9. Show that the iteration formula for finding the reciprocal of N is $x_{n+1} = x_n(2 - N_{xn}),\ \ n = 0,1,\ldots.$ **[1997]**

Q10. Use Regula-Falsi method to show that the real root of $x\log_{10} x - 1.2 = 0$ lies between 3 and 2.740646. **[1998]**

Q11. Find the positive root of the equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$. Using Newton-Raphson method correct to 4 decimal places. Also show that the following scheme has error of second order

$$x_{n+1} = \frac{1}{2}x_n\left(1 + \frac{a}{x_n^2}\right).$$

[2003]

Q12. How many positive and negative roots of the equation $e^x - 5\sin x = 0$ exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method. **[2004]**

Q13. Use the method of false position to find a real root of $x^3 - 5x - 7 = 0$ lying between 2 and 3 and correct to 3 places of decimals. **[2007]**

Q14. Develop at algorithm for Regula-Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the rot such that $f(x_0) \neq f(x_1)$. Take n as the maximum number of iteration allowed and eps be the prescribed error. **[2010]**

Q15. Find the positive root of the equation $10xe^{-x^2} - 1 = 0$ correct upto 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[2010]**

Q16. Use Newton-Raphson method to find the real root of $3x = \cos x + 1$ correct to 4 decimal places. **[2012]**

Q17. Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$. **[2013]**

Q18. Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct upto 4 decimal places. **[2014]**

Q19. Write an algorithm in the form of a flowchart for Newton Raphson method. Describe the cases of failure of this method. **[2017]**

Q20. Write down the basic algorithm for solving the equation $xe^x - 1 = 0$ by insection method, correct to 4 decimal places. **[2018]**

Q21. Apply Newton Raphson method to find a real root of transcendental equation $x\,log_{10}x = 1.2$ correct to 3 decimal places. **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 24: Interpolation

Q1. Certain corresponding values of x and $\log x$ are given

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4847

Find $\log_{10} 301$, using Lagrange's interpolation formula. **[1988]**

Q2. Apply Lagrange's formula to find a root of the equation $f(x)=0$ given that

$f(30)=-30,\quad f(34)=-13,\quad f(38)=3,\quad f(42)=18.$ **[1991]**

Q3.The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the condition of an electric current.

t	1.2	2.0	2.5	3.0
i	1.36	0.58	0.34	0.20

Applying an appropriate formula interpolate for the value of i when $t=1.6$.

[1996]

Q4. Find the cubic polynomial which takes following values:

$y(0)=1,\quad y(1)=0,\quad y(2)=1$ & $y(3)=10$

Hence, otherwise $y(4)$. **[2002]**

Q5. Find the unique polynomial $P(x)$ of degree 2 or less such tha $P(1)=1,\ P(3)=27,\ P(4)=64$. Using the Lagrange's interpolation formula and the Newton's divided difference formula evaluate $P(15)$.

[2005]

Q6. The following values of the function $f(x)=\sin x+\cos x$ are given

x:	10	20	30
$f(x)$:	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate $f\left(\frac{\pi}{12}\right)$. Compare with exact value.

Q7. Value of $f(3)$ from the following table:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

[2009]

Q8. Draw a flow chart for Lagrange's interpolation.

Q9. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. **[2013]**

Q10. Find the Lagrange interpolating polynomial that fits the following data

x : -1 2 3 4

$f(x)$: -1 11 31 69

find $f(1.5)$ **[2015]**

Q11. Let $f(x) = e^{2x}$ Cos $3x$ for $x \in [0, 1]$. Estimate the value of $f(0.5)$ using lagrange interpolating polynomial of degree 3 over the nodes $x = 0$, $x = 0.3$, $x = 0.6$ and $x = 1$. Also compute the ever bound over the interval [0, 1] and the actual ever $\in (0.5)$ **[2016]**

Q12. For given equidistant values u_{-1} , u_0 , u_1 *and* u_2 a value is interpolated by lagrang's formula. Show that it may be written in the form.

$$u_x = yu_o + xu_1 + \frac{y(y^2-1)}{3!} \quad \Delta^2 u = 1$$

$$+\frac{x(x^2-1)}{3!}\,\Delta^2 u_o \text{ , where } x + y = 1$$ **[2017]**

Q13. Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1$, $u_2 = 9$, $u_3 = 25$, $u_4 = 55$ *and* $u_5 = 105$ **[2018]**

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 25: Gauss Elimination/Gauss seidel method

Q1. Solve the following system of linear equations using Gauss elimination method:

$$x_1 + 6x_2 + 3x_3 = 6$$
$$2x_1 + 3x_2 + 3x_3 = 117$$
$$4x_1 + x_2 + 2x_3 = 283.$$

[2000]

Q2. Using Gauss-Seidel iterative method and the starting solution as $x_1 - x_2 - x_3 = 0$, determine the solution of the following system of equations in two iterations:

$$10x_1 - x_2 - x_3 = 8$$
$$x_1 + 10x_2 + x_3 = 12$$
$$x_1 - x_2 + 10x_3 = 10$$

Q3. Using Gauss-Seidel iteration method, find the solution of the following system upto three iteration:

$$4x - y + 8z = 26$$
$$5x + 2y - z = 6$$
$$x - 10y + 2z = -13.$$

[2004]

Q4. Apply Gauss-Seidel method to calculate x, y, z from the system

$$-x - y + 6z = 42$$
$$6x - y - z = 11.33$$
$$-x + 6y - z = 32$$

with initial values (4.67, 7.62, 9.05). Carry out computations for two iterations.

[2008]

Q5. Solve using Gauss-Seidel method:

$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25.$$

[2012]

Q6. Solve the system of equations

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$

Using Gauss-Seidel iteration method (perform three iteration). **[2014]**

Q7. Find the solution of the system

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10\,x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 - 10x_4 = -9$

Using Gauss Seidel method (make 4 iterations) **[2015]**

Q8. Explain the main steps of the Gauss Jordan method and apply this method to find the inverse of the matrix

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 8 & 8 \end{bmatrix}$$

[2017]

Q9. Apply Gauss Seidel iteration method to solve the following system of equations

$$2x + y - 2z = 17$$

$$3x + 20y - z = 18$$

$$2x - 3y + 20z = 25$$

Correct to 3 decimal place **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 26: Numerical Integration

Q1. Use the method of Gauss to verify the following computation

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1+\sin^2\theta}} = 1.311028.$$ **[1990]**

Q2. Evaluate approximately $\int_{-3}^{3} x^4 dx$ by Simpson's rule by taking seven equidistant ordinates. Compare it with the value obtained by trapezoidal rule and with exact value. **[1993]**

Q3. Find the value of $\int_{1.6}^{3.4} f(x)dx$ from the following data using Simpson's 3/8th rule for the interval (1.6, 2.2) and 1/3rd rule for (2.2, 3.4)

x	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
$f(x)$	4.953	6.050	7.389	9.025	11.023	13.464	16.445	20.086	24.533	29.964

Q4. Evaluate $\int_1^3 \frac{dx}{x}$ by Simpson's rule with 4 strips. Determine the error by direct integration.

Q5. Obtain Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \le 3$. Show that the error of approximation for Simpson's rule is given by

$$R = -\frac{(b-a)}{2880} f^{iv}(\eta) \quad \eta \in (0,2)$$

Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \le 0.008333$. **[1999]**

Q6. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by subdividing the interval $(0,1)$ into 6 equal parts and using Simpson's 1/3rd rule. Hence find the value of π and actual error, correct to five places of decimal. **[2000]**

Q7. Evaluate $\int_0^1 e^{-x^2} dx$ employing three point Gaussian quadrature formula, finding the required weights and residues. Use five decimals for computation. **[2003]**

Q8. Draw a flow chart and write a program in basic for Simpson's 1/3rd rule for integration $\int_a^b \frac{dx}{1+x^2}$ correct upto 10^{-6}. **[2003]**

Q9. The velocity of a particle at distances from a point on its path is given by the following table:

S meter	0	10	20	30	40	50	60
V m/sec	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's 1/3rd rule. Compare the result with Simpson's 3/8th rule. **[2004]**

Q10. Use appropriate quadrature formula out of Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1+x^2}$ with $h=0.2$. Hence obtain an approximate value of π. Justify the use of particular quadrature formula. **[2005]**

Q11. Evaluate $\int_0^1 e^{-x^2} dx$ by Simpson's 1/3rd rule with $2\pi=10,\ x_0=0,\ h=0.1,\ x_{10}=1.0$.

[2006]

Q12. Find from the following table, the area bounded by the x- axis and the curve $y=f(x)$ between $x=5.34$ and $x=5.4$ using Trapezoidal rule.

x	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

Q13. Evaluate $\int_1^5 \log_{10} x dx$ by Simpson's 1/3rd rule correct upto 4 decimal places. Take 8 sub intervals in your computation. **[2010]**

Q14. A solid of revolution is formed by rotating about the x- axis, the area between the x- axis, the line $x=0$ and $x=1$ and a curve through the points with the following coordinates:

x	0.0	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Find value of the solid. **[2011]**

Q15. The velocity of a train which starts from rest given in the following table

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using Simpson's 1/3rd formula

[2013]

Q16. Use five subintervals to integrate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule. **[2014]**

Q17. For an integral $\int_{-1}^{1} f(x)\ dx$ show that the two point Gauss quadrature rule is given by

$\int_{-1}^{1} f(x)dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(\frac{-1}{\sqrt{3}}\right)$ Using this rule estimate $\int_2^4 2xe^x dx$ **[2016]**

Q18. Derive the formula

$$\int_a^b y\,dx = \frac{3h}{8}[(y_o + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots + y_{n-1}) + 2(y_3 + y_6 + \cdots + y_{n-3})]$$

Is there any restriction on n? State the condition what is the error bound in case of simpson's $\frac{3}{8}$ rule.

Q19.

time (minutes)	2	4	6	8	10	12	14	16	18	20
speed (km/hr)	10	18	25	29	32	20	11	5	2	8.5

Starting from the rest in the beginning speed (in km/hr) of a train at different times (in minutes) is given by the above table.

Using Simpson's $\frac{1}{3}$ rd rule, find the approximate distance travelled (in km) in 20 minutes from the beginning. **[2018]**

Q20. Find values of the constant a, b, c such that the quadrature formula

$\int_o^h f(x)dx = h\left[af(o) + bf\left(\frac{h}{3}\right) + cf(h)\right]$

Is exact for polynomials of as high degree as possible and hence find the order of translation even. **[2018]**

Q21. Draw a flowchart and write a basic algorithm in (FORTAN/C/C++)

For calculating $y = \int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 27: Numerical Solution of Ordinary Differential Equation

Q1. Use Runge-Kutta method to solve

$$10\frac{dy}{dx} = x^2 + y^2, \qquad y(0) = 1$$

for the interval $0 < x \le 0.4$ with $h = 0.1$. **[1989]**

Q2. Using Runge-Kutta method with 3rd order accuracy solve $\frac{dy}{dx} = y - x$ with initial condition $y = 2,\ x = 0$.

[1990]

Q3. Solve by Runge-Kutta method $\frac{dy}{dx} = x + y$ with the initial conditions $x_0 = 0$, $y_0 = 1$ correct upto 4 decimal places by evaluating upto second increment of y. (Take $h = 0.1$).

[1992]

Q4. Solve $\frac{dy}{dx} = xy$ for $x = 1.4$ by Runge-Kutta method, initially $x = 1,\ y = 2$. (Take $h = 0.2$). **[1993]**

Q5. Apply that 4th order Runge-Kutta method to find value of y correct to 4 places of decimals at $x = 0.2$, when $y' = \frac{dy}{dx} = x + y,\ y(0) = 1$.

[1997]

Q6. By the 4th order Runge-Kutta method, tabulate the solution of Differential Equation

$$\frac{dy}{dx} = \frac{xy + 1}{10y^2 + 4}, \qquad y(0) = 0$$

in [0, 0.4] with step length of 0.1 correct to 5 places of decimal. **[1998]**

Q7. Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ and $x = 0$, find $y(0.2)$, $y(0.4)$ and $y(0.6)$.

Q8. Using 4th order classical Runge-Kutta method for the initial value problem

$$\frac{du}{dt} = -2tu^2 \qquad u(0) = 1$$

with $h = 0.02$ on the interval [0, 1]. Calculate $u(0.4)$ correct to six places of decimal.

[1999]

Q9. Runge-Kutta 4th order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution. $e^{0.1} = 1.10517,\ e^{0.2} = 1.2214$. **[2002]**

Q10. Apply the 2nd order Runge-Kutta method to find an approximate value of y at $x = 0.2$ taking $h = 0.1$ given that y satisfies the Differential Equation $y' = x + 1$ with initial condition $y(0) = 1$.

[2007]

Q11. Find the value of $y(1.2)$ using Runge-Kutta 4th order method with step size $h = 0.2$ from the initial value problem $y' = xy,\ y(1) = 2$. **[2009]**

Q12. Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data

x	0.1	0.2	0.3	0.4
y	0.9975	0.9900	0.9776	0.9604

[2012]

Q13. Provide a computer algorithm to solve an O.D.E. by Euler's method to solve $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$. **[2012]**

Q14. Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct upto 5 decimal places for

$$y' = x(y + x) - 1, \qquad y(0) = 2$$ **[2013]**

Q15. Use Runge-Kutta formula of 4th order to find the value of y at $x = 0.8$, where $\frac{dy}{dx} = \sqrt{x + y}$, $y(0.4) = 0.41$. Take the step length $h = 0.2$. **[2014]**

Q16. Solve the initial value problem $\frac{dy}{dx} = x(y - x),\ y(2) = 3$ in the interval [2, 2, 4] using the Runge Kutta fourth order method with step size $h = 0.2$ **[2015]**

Q17. Using Runge Kutta method of 4th order, Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $= 0.2$. Use four decimal places for calculation and step length 0.2. **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 28: Number Systems/Boolean Algebra

Q1. (i) Covert the following binary number into octal and hexadecimal system

1 0 1 1 1 0 0 1 01 0 0 1 0

(ii) Find the multiplication of the following binary numbers

1 1 0 0 1 1 and 1 0 1 ·1 **[2003]**

Q2. (i) Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011\cdot 10)_2$ **[2005]**

(ii) Given the number 59.625 in decimal system. Write its binary equivalent.

(iii) Given the number 3898 in decimal system. Write its equivalent in system base 8.

[2006]

Q3. (i) Covert $(46655)_{10}$ into one in base 6.

(ii) $(11110\cdot 01)_2$ into a number in the decimal system. **[2007]**

Q4. Find the values of two valued Boolean variables A, B, C, D by solving the following simultaneous equations:

$$\bar{A} + AB = 0$$
$$AB = AC$$
$$AB + A\bar{C} + CD = C\bar{D}.$$ **[2009]**

Q5.(i) Realize the following expression by using NAND gates only $g = (\bar{a} + \bar{b} + c)\bar{d}(\bar{a} + e) f$

(ii) Find the decimal equivalent of $(357\cdot 32)_8$. **[2009]**

Q6. If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. **[2010]**

Q7. (i) Find the hexadecimal equivalent of the decimal $(587632)_{10}$.

(ii) Simplify the following:

(a) $a + a'b + a'b'c + a'b'c'd + ...$

(b) $x'y'z + yz + xz$

(iii) For the given set of data points $(x_1, f(x_1)), (x_2 f(x_2)),....(x_n f(x_n))$, write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula.

[2010]

(iv) Compute $(3205)_{10}$ to base 8. **[2011]**

Q8. Draw a flow chart for Lagrange's interpolation formula. **[2011]**

Q9. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Q10. Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x,y)$ in the interval $[a,b]$ for n number of discrete points, where the initial value is $y(a)=\alpha$ using Euler's method.

[2012]

Q11. Develop an algorithm for Newton-Raphson method to solve $f(x)=0$ starting with initial iterate x_0, n be the number iterations allowed, $\in$ be the prescribed error and δ be the prescribed lower bound for $f(x)$. **[2013]**

Q12. Draw a flowchart for Simpson's 1/3rd rule. **[2014]**

Q13. For any Boolean variables x and y, show that $x+xy=x$. **[2014]**

Q14. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z=xy+uv$.

[2014]

Q15. Find the principal (or canonical) disjunctive normal form in three variables p, q, r for the Boolean algebra $((p \cap q) \rightarrow r > v((p \cap q) \rightarrow -r)$. Is the given Boolean expression a contradiction or a tautology?

Q16. Convert the following decimal numbers to equivalent binary and hexadecimal numbers

(1) 4096

(2) 0.4375

(3) 2048.0625 **[2016]**

Q17. Let A, B, C be Boolean variables A denote complement of A, A+B is an expression for A or B and A B is an expression for A AND B. Then simply the following expressions & draw a block diagram of the simplified expression using A N D and O R gates

$A(A + B + C).(\overline{A} + B + C).\left(A + \overline{B} + C\right).(A + B + \overline{C})$ **[2016]**

Q18. Write the Boolean expression

$z(y + z)\ (x + y + z)$ in its simplest form using Boolean postulates rules. Mention the rules used during simplification verify your result by constructing the truth table for the given expression and for its simplest form. **[2017]**

Q19. Find the equivalent of numbers given in a specified number system to the system mentioned against them

(1) $(1111011.101)_2$ to decimal system
(2) $(1000111110000.00101100)_2$ to hexadecimal system
(3) $(C4\ F2)_{16}$ to decimal system
(4) $(418)_{10}$ to binary system **[2018]**

Q20. Simply the Boolean expression

$(a + b).\left(\overline{b} + c\right) + b.(\overline{a} + \overline{c})$ by using the laws of Boolean algebra. From its truth table write it in minterm normal form. **[2018]**

Q21. Find the equivalent number given in a specified number to the system mentioned against them

(1) Integer 529 in binary system
(2) 101010110101.101101011 to octal system
(3) Decimal number 5280 to hexadecimal system
(4) Find the unknown number $(1101.101)_8 \rightarrow (?)_{10}$ **[2019]**

Q22. Given the Boolean expression

$X = AB + ABC + A\overline{B}\,\overline{C} + A\overline{C}$

(i) Draw the logical diagram for the expression
(ii) Minimize the expression
(iii) Draw the logical diagram for the reduced expression **[2019]**

CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Fluid Dynamics

TUTORIAL SHEET 29: KINEMATICS

Q1. If the velocity distribution of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{kz^2r^2}{r^5}\right)$ then determine the parameter k, such that it is a possible fluid motion. Hence find its velocity potential. **[2001]**

Q2. Show that the velocity potential $\phi = \frac{1}{2}a\left(x^2 + y^2 - 2z^2\right)$ satisfies the Laplace equation and determine the stream lines. **[2002]**

Q3. For an incompressible homogeneous fluid at the point (x, y, z) the velocity distribution is given by $u = \frac{c^2 y}{r^2}, v = \frac{c^2 x}{r^2}, \omega = 0$, where r denotes distance from Z-axis. Show that it is a possible fluid motion and determine the surface which is orthogonal to stream lines.

[2003]

Q4. If the velocity potential of a fluid is $\phi = \frac{z}{r^3}\tan^{-1}\frac{y}{z}$ then show that stream lines lie on the surfaces $x^2 + y^2 + z^2 = c\left(x^2 + y^2\right)^{\frac{2}{3}}$, c being constant. **[2008]**

Q5. Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $q \to 0$ as $r \to \infty$, find the surface of constant speed.

[2012]

Q6. If the velocity of an incompressible fluid at the point (x , y ,z) is given by

$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$, $r^2 = x^2 + y^2 + z^2$

Then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further determine the streamline. **[2017]**

Q7. For an incompressible fluid flow, two components of velocity (u ,v ,w) are given by

$u = x^2 + 2y^2 + 3z^2$, $v = x^2y - y^2z + zx$

Determine the third component w so that they satisfy equation of continuity. Also find the z component of acceleration. **[2018]**

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 30: EULER'S EQUATION OF MOTION

Q1. A infinite mass of fluid is acted upon by a force $\mu r^{-3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of a sphere $r = c$ in it; show that cavity will be filled up after an interval of time $\left\{\frac{2}{5\mu}\right\}^{\frac{1}{2}} c^{\frac{5}{4}}$. **[2003, 2009]**

Q2. State the condition under which Euler's equation of motion can be integrated. Show that $\frac{-\partial\phi}{\partial t} + \frac{q^2}{2} + v + \int \frac{dP}{P} = F(t)$, where symbols have their usual meaning. **[2005]**

Q3. Liquid is contained between two parallel planes, the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated. Prove that if P be the pressure at the outer surface, the initial pressure at any point of the liquid distance r from the centre is $P\frac{\log r - \log b}{\log a - \log b}$ **[2006]**

Q4. An infinite mass of homogeneous incompressible fluid is at rest subject to a uniform pressure π and contains a spherical cavity of radius a filled with gas at a pressure $m\pi$. Prove that if the inertia of the gas is neglected and Boyle's law be supposed to hold through the insuling motion, the radius of the sphere will oscillate between the values a and na, where n is determined by the equation $1 + 3m\log n - n^3 = 0$. If m be nearly equal to 1, the time of oscillation will be $2\pi\sqrt{\frac{\rho a^2}{3\pi}}$, where ρ being the density of fluid.

Q5. An infinite fluid which is contained in a spherical hollow region of radius a is initially at rest under the action of no forces. If a constant pressure P is applied at infinity, show that the time of filling up the cavity is $\pi^2 a\left(\frac{\rho}{\pi}\right)^{1/2} 2^{5/6}\left(\left|\frac{1}{3}\right)^{-3}$.

Q6. Air, obeying Boyle's law is in motion in a uniform tube of small section. Prove that if ρ be the density and v velocity at a distance x from a fixed point at time t, then

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left\{\rho\left(v^2 + k\right)\right\}.$$

Q7. A quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore under the action of a force to a point in the tube varying as a distance from that point. Determine the motion and pressure.

Q8. A steady inviscid incompressible blow has a velocity field $u = fx,\ v = -fy$ and $\omega = 0$, where f is any constant. Derive an expression for the pressure field $P(x, y, z)$, if the pressure $P(0,0,0) = P_0$ and $\vec{g} = -g\hat{k}$.

[2006]

Q9. A steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to be steady and diverging from the vortex of the cone, then prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2K}$$

Where K is the pressure divided by the density and is constant. **[2017]**

Q10. A sphere of radius R , whose centre is at rest , vibrates radially in an infinite incompressible fluid of density ρ , which is at rest at infinity. If the pressure at infinity is , so that the pressure at the surface of the sphere at time t is $\pi + \frac{1}{2}\rho\{\frac{d^2R^2}{dt^2} + (\frac{dR}{dt})^2\}$

[2019]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 31: Potential Flow, Motion in 2 Dimensions

Q1. If complex potential $\omega = Az^2$, find velocity potential, stream lines and velocity at origin.

Q2. If $z = c\cos\omega$ and $z = c\cosh\omega$, find ϕ and ψ.

Q3. Find the lines of flow in 2 D given by $\omega = \phi + i\psi = -\frac{1}{2}n(x+iy)^2 e^{2int}$. Prove the paths of the particles of the fluid may be obtained by eliminating t from the equations

$$r\cos(nt+0) - x_0 = r\sin(nt+\theta) - y_0 = nt(x_0 - y_0)$$

Q4. If a homogeneous liquid is acted on by a repulsive force from the origin, the magnitude of which at a distance r from the origin is μr per unit mass, show that it is possible for the liquid to move steadily without being constrained by any boundaries in the space between one branch of the hyperbola $x^2 - y^2 = a^2$ and the asymptotes and find the velocity potential

Q5. What arrangement of sources and sinks will give rise to the function $\omega = \log\left(z - \frac{a^2}{z}\right)$ .Prove that two of the stream lines subdivide into the circle $r = a$ and the axis of Y .

Q6. In the region bounded by a fixed quadrilateral arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of the bounding radii. Show that streamlines leaving at either end at angle α with the radius is $r^2\sin(\alpha+\theta) = a^2\sin(\alpha-\theta)$.

Q7. In the 2 D motion of an infinite liquid there is a rigid boundary consisting of that part of the circle $x^2 + y^2 = a^2$, which lies in the first and fourth quadrants simple source of strength m is placed at the point $(f,0)$ where $f > a$. Prove that the speed of the fluid at the point $(a\cos\theta, a\sin\theta)$ of the semi-circular boundary is $\frac{4a\,mf^2 2in2\theta}{a^4 + f^4 - 2a^2 f^2\cos 2\theta}$. Find at what point of the boundary the pressure is least.

Q8. Two sources of strength m are placed at the points $(-a,0),(a,0)$ and a sink of strength 2m at the origin. Show that the stream lines are the curves $(x^2+y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is any variable parameter. Show also that the fluid speed at any point is $\frac{2ma^2}{(r_1 r_2 r_3)}$ where r_1 , r_2 and r_3 are the distances of the points from the sources and sinks respectively.

[1999,2019]

Q9. Between the fixed boundaries $\theta = \frac{\pi}{4}$ and $\theta = \frac{-\pi}{4}$ there is a 2 D liquid motion due to a source of strength m at the point $(r = a, \theta = 0)$ and an equal sink at the point $(r = b, \theta = 0)$. Show that the stream function is $-m\tan^{-1}\frac{r^4(a^4 - b^4)\sin 4\theta}{r^8 - (a^4 + b^4)r^4\cos 4\theta + a^4b^4}$ and show that the velocity at (r, θ) is

$$\frac{4m(a^4 - b^4)r^3}{(r^8 - 2a^4r^4\cos 4\theta + a^8)^{1/2}(r^3 - 2b^4r^4\cos 4\theta + b^8)^{1/4}}.$$ **[1994,1998]**

Q10. If fluid fills the region of space on positive side of the x- axis, which is a rigid boundary and if there be a source m at $(a, 0)$ and an equal sink at $(0, b)$ and if the pressure on negative side be the same as at ∞. Show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{2ab(a+b)}$.

[1995, 2008, 2012]

Q11. Show that the velocity potential $\phi = \frac{1}{2}\log\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}$ gives a possible fluid motion. Determine the stream lines and show also that the curves of equal speed are the ovals of cassini given by $rr' =$ constant. **[2014]**

Q12. A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity U $\vec{i}$. Show that velocity potential $\emptyset$ at any point P of the stream is $\frac{m}{r} - Ur\ cos\theta$, where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{i}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2\ Sin^2\theta - 2m\cos\theta =$ constant. **[2016]**

Q13. For a two dimensional potential flow the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}(x^3 - y^3)$. Determine the velocity components along the directions x and y. Also determine the stream function ψ and check whether ϕ represent a possible case of flow or not.

[2018]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 32: Axisymmetric Motion

Q1. Find velocity potential & stream function at any point of liquid contained between two coaxial cylinders of radii a & $b(a<b)$ when the cylinders are moved suddenly parallel to themselves in directions at right angles with velocities u & v respectively.

Q2. Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points. **[2015]**

Q3. In an axisymmetric motion, show that stream function exists due to equation of continuity. Express the velocity components in terms of the stream function. Find the equation satisfied by the stream function if the flow is irrotational.

[2015]

Q4. The Space between two concentric spherical shells of radii a, b (a < b) is filled with a liquid of density p. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential

$$\emptyset = \frac{\left\{a^3U\left[1+\frac{1}{2}b^3r^{-3}\right]x - b^3V\left[1+\frac{1}{2}a^3r^{-3}\right]y\right\}}{(b^3-a^3)}$$

where $r^2 = x^2 + y^2 + z^2$, the coordinates being rectangular. Evaluate the velocity at any point of the liquid. **[2016]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 33: Vortex Motion

Q1. In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Do the velocity components satisfy the Laplace equation? Justify. **[2004]**

Q2. In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Show that the components of velocity u, v, ω are solutions of Laplace equation. **[2010]**

Q3. When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distance from its axis. Show that the path of each vortex is given by

$\left(r^2 \sin^2\theta - b^2\right)\left(r^2 - a^2\right)^2 = 4a^2b^2r^2\sin^2\theta$, θ being measured from the line through the centre perpendicular to the joint of the vortices. **[2010]**

Q4. An infinite row of equidistant rectilinear vortices are at a distance "a" apart. The vortices are of the same strength k but are alternatively of opposite signs. Find the complex function that determines the velocity potential and the stream function. **[2011]**

Q5. If n rectilinear vertices of the same strength k are symmetrically arranged as generators of a circular cylinder of radius "a" in an infinite liquid. Prove that vortices will move round the cylinder uniformly in time $\dfrac{8\pi^2 a^2}{(n-1)k}$. Find the velocity at any point of the liquid. **[2013]**

Q6. Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the streamlines are $(u, v, \omega) = \mu\left(\dfrac{\partial\phi}{\partial x}, \dfrac{\partial\phi}{\partial y}, \dfrac{\partial\phi}{\partial z}\right)$, where μ and ϕ are functions of (x, y, z, t). **[2013]**

Q7. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$

Possess vorticity , where $\vec{q}$ (u , v , w) is the velocity in the Cartesian frame , $\vec{r}$ =(x , y , z) and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle$x^2 + y^2 = 9$, z=0 ? **[2017]**

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 34: N-S Equations

Q1. Find Navier-Stokes equation for a steady laminar flow of a viscous incompressible fluid between two infinite parallel plates. **[2014]**

CIVIL SERVICE EXAMINIATION (MAINS)

Mathematics Paper II: Mechanics

TUTORIAL SHEET 35: Moments of Inertia

Q1. Find the M.I of a solid hemisphere about a diameter of its plane base. **[1999]**

Q2. Find the M.I. of an elliptic area about a line CP inclined at θ to the major axis and about a tangent parallel to CP, where C is the centre of ellipse. **[2000]**

Q3. Determine the M.I of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and through centre of its base **[2001]**

Q4. Find the M.I of a circular wire about (i) a diameter and (ii) a line through the centre and perpendicular to its plane. **[2002]**

Q5. A solid body of density ρ is in the shape of the solid formed by the revolution of the cardiod $r = a(1+\cos\theta)$ about the initial line. Show that its M.I about the straight line through the pole and perpendicular to the initial line is $\frac{352}{105}\pi\rho a^2$.

[2003]

Q6. The flat surface of a hemisphere of radius r is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be 'l' and the total mass be 'm', show that the M.I of the combination about the axis of cylinder is

$$mr^2 \frac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$$ **[2009]**

Q7. Let 'a' be the radius of the base of a right circular cone of height h and mass M. Find the M.I of that right circular cone about a line through the vertex perpendicular to the axis. **[2011]**

Q8. A pendulum consists of a rod of length $2a$ and mass m to one end of which a spherical bob of radius $\frac{a}{3}$ and mass 15 m is attached. Find the M.I of the pendulum:

(i) about an axis through the other end of the rod and at right angles to the rod

(ii) about a parallel axis through the centre of mass of the pendulum. [Given the centre of mass of the pendulum is $\frac{a}{12}$ above the centre of sphere .

[Given : the centre of mass of the pendulum is a/12 above the centre of the sphere]

[2012]

Q9. Four solid sphere A, B, C and D each of the mass m and radius "a", are placed with their centres on the four corners of a square of side "b". Calculate the M.I of the system about a diagonal of the square.

[2013]

Q10. Calculate the moment of inertia of a solid uniform hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$ with mass m about the OZ – axis. **[2015]**

Q11. Show that the moment of inertia of an elliptic area of mass M and semi axis a and b about a semi diameter of length r is $\frac{1}{4} m \frac{a^2 b^2}{r^2}$. Further , prove that the moment of inertia about a tangent is

$\frac{5M}{4} p^2$, where p is the perpendicular distance from the the centre of the ellipse to the tangent.

[2017]

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 36: Lagrange's Equation of Motion

Q1. How do you characterize

i) The simplest dynamical system?

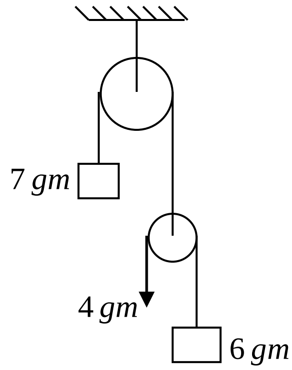
ii) The most general dynamical system?

Show that equation of motion $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)$- $\frac{dT}{\partial q_k} = Q_k$ correspond to non conservative but sceleronomic and holonomic system with n degrees of freedom.

[1995]

Q2. Six equal uniform rods form a regular hexagon loosely formed at the angular points and rest on smooth table. A blow is given perpendicular to one of them at its middle point. Show that the opposite rod begins to move with one tenth of the velocity of the rod that is struck. **[1996]**

Q3. A pulley system is given as shown in diagram. Discuss the motion of the system, using Lagrange's method the pulley wheels have negligible masses and moment of inertia and three wheels are frictionless. **[1997]**



Q4. Using Lagrange's equations obtain the differential equation of planetary motion **[1997]**

Q5. Using Lagrange's equation, obtain the differential equation of motion of a free particle in spherical polar coordinates. **[1998]**

Q6. Two particles in a plane are connected by a rod of length and are constrained to move in such a manner the velocity of the middle of the rod is in the direction of rod. Write down the equation of constraints. Is the system holonomic or non-holonomic. Give reasons for your answer. **[1998]**

Q7. A particle of mass m moves in space with Lagrengian $L=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-V+\dot{x}A+\dot{y}B+\dot{z}C$, where V, A are given functions of x, y, z.

Show that the equation of motion are $m\ddot{x}=-\frac{\partial V}{\partial x}+\dot{y}\left[\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}\right]-\dot{z}\left[\frac{\partial A}{\partial z}-\frac{\partial C}{\partial x}\right]$ and two similar equations for y and z. Find also the Hamiltonian H in terms of generalized momenta.

[1997]

Q8. Find the equation of motion for a particle of mass m which is constrained to move on the surface of a cone of Semi-Vertical angle α and which is subjected to a gravitational force.

[2001]

Q9. Obtain the equations governing the motion of a spherical pendulum. **[2012]**

Q10. Suppose the Lagrangian of a mechanical system is given by

$$L=\frac{m}{2}(a\dot{x}^2+2b\dot{x}\dot{y}+c\dot{y}^2)-\frac{K}{2}(ax^2+2bxy+cy^2)$$

Where $a, b, c, m>0$, $K>0$ are constants and $b^2\neq ac$. Write down the lagrangian of equations of motion and identify the system. **[2018]**

Q11. A hoop with radius r is rolling, without slipping , down an inclined plane of length l and with angle of inclination $\emptyset$. Assign appropriate generalized coordinates to the system. Determine the constraints, if any. Write down the Lagrangian equations for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane.

[2016]

Q12. To uniform rods AB, AC, each of mass m and length 2a are smoothly hinged together at A and move on a horizontal plane. At time t, the mas centre of the rods is at the point $(\xi\,,\eta)$ referred to fixed perpendicular axes Ox, Oy in the plane and the rods make angles $\theta\pm\emptyset$ with Ox. Prove that kinetic energy of the system is

$$m\left[\dot{\xi}^2+\dot{\eta}^2+\left(\frac{1}{3}+sin^2\emptyset\right)a^2\dot{\theta}^2+\left(\frac{1}{3}+cos^2\emptyset\right)a^2\emptyset^2\right]$$

Also derive Lagrange's equations of motion for the system if an external force with components [X, Y] along the axes acts of A.

[2017]

CIVIL SERVICE EXAMINIATION (MAINS)

TUTORIAL SHEET 37: Hamilton's Equation of Motion

Q1. Derive the Hamilton equations of motion from the principle of least action and obtain the same for a particle of mass m moving in a force field of potential V. Write these equations in spherical coordinates (r, θ, ϕ).

[2004]

Q2. A particle of mass m is constrained to move on the surface of a cylinder. The particle is subjected to a force directed towards the origin and proportional to the distance of the particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion. **[2006]**

Q3. A point mass m is placed on a frictionless plane that is tangent to the earth's surface. Determine Hamilton's equations by taking x or θ as the generalized coordinate. **[2007]**

Q4. A sphere of radius 'a' and mass M rolls down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[2010]**

Q5. Find the equation of motion of a compound pendulum using Hamilton's equations.

[2014]

. Q6. Consider a single free particle of mass m, moving in space under no forces. If the particle starts from the origin at $t = 0$. and reaches the position (x, y, z) at tine τ, find the Hamilton's characteristic function S as a function of x, y, z, τ. **[2016]**

Q7. The Hamiltonian of a mechanical system is given by

$$H = p_1 q_1 - a{q_1}^2 + b{q_2}^2 - p_2 q_2$$

Where a, b are the constants . Solve the Hamiltonian equation and show that

$\frac{p_2 - bq_2}{q_1} = constant$ **[2018]**

Q8. Using Hamilton's equation, find the acceleration for a sphere rolling down a rough inclined plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane

[2019]

CIVIL SERVICE EXAMINIATION (MAINS)
TUTORIAL SHEET 38: Equation of Motion in 2 D / De Alembert's Principle

Q1. A uniform rod OA of length $2a$ free to turn about its end O revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either O or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

[1996]

Q2. A planet of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizontal and a man of mass M. Starting from the upper end walks down the plank. Show that it does not move. Show that he gets to other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$, where a is length of the plank **[2000, 2005]**

Q3.A uniform rod of mass $3m$ and length $2l$ has its middle point fixed and a mass m is attached to one of its extremity. The rod, when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity $\sqrt{\frac{28}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $(\sqrt{2}-1)$ **[2008]**

Q4. A perfectly rough sphere of mass m and radius b rests on the lowest point of a fixed spherical cavity of radius 'a'. To the highest point of a movable sphere is attached a particle of mass 'M' and the system is disturbed. Show that the oscillations are the same as those of simple pendulum of length

(a-b)$\frac{4m'+\frac{7}{5}m}{m+m'(2-\frac{a}{b})}$ **[2009]**

Q5. A circular cylinder of radius a and radius of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane through axis moves in circular pendulum of length $(b-a)\left(1+\frac{K^2}{a^2}\right)$. **[2019]**

Q6.) A uniform rod OA , of length 2a , free to turn about its end O, revolves with angular velocity ω about the vertical OZ through O ,and its inclined at its constant angle α to OZ ; find the value of α **[2019]**